
Classical results.

1. **AM-GM.** For any non-negative real numbers $x_1, x_2, \ldots, x_n$,

   \[
   \sqrt[n]{x_1 x_2 \cdots x_n} \leq \frac{x_1 + x_2 + \cdots + x_n}{n}.
   \]

2. **Cauchy-Schwarz.** For any real $x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_n$,

   \[
   (x_1 y_1 + x_2 y_2 + \cdots + x_n y_n)^2 \leq (x_1^2 + x_2^2 + \cdots + x_n^2)(y_1^2 + y_2^2 + \cdots + y_n^2).
   \]

3. **Jensen.** For any convex function $f$ and any real $x_1, x_2, \ldots, x_n$,

   \[
   f \left( \frac{x_1 + x_2 + \cdots + x_n}{n} \right) \leq \frac{f(x_1) + f(x_2) + \cdots + f(x_n)}{n}.
   \]

Problems.

1. Show that

   \[
   \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \cdots \frac{2n-1}{2n} < \frac{1}{\sqrt{2n+1}}.
   \]

2. **Putnam 2003. A2.** Let $a_1, a_2, \ldots, a_n$ and $b_1, b_2, \ldots, b_n$ be nonnegative real numbers. Show that

   \[
   (a_1 a_2 \cdots a_n)^{1/n} + (b_1 b_2 \cdots b_n)^{1/n} \leq [(a_1 + b_1)(a_2 + b_2) \cdots (a_n + b_n)]^{1/n}.
   \]

3. **Putnam 2004. B2.** Let $m$ and $n$ be positive integers. Show that

   \[
   \frac{(m+n)!}{(m+n)^{m+n}} < \frac{m! n!}{m^m n^n}.
   \]

4. **USA 1997.** A set of $n > 3$ real numbers has sum at least $n$ and the sum of the squares of the numbers is at least $n^2$. Show that the largest positive number is at least 2.

5. **IMO 1994.** Let $m$ and $n$ be positive integers. Let $a_1, a_2, \ldots, a_m$ be distinct elements of $\{1, 2, \ldots, n\}$ such that whenever $a_i + a_j \leq n$ for some $i, j$ (possibly the same) we have $a_i + a_j = a_k$ for some $k$. Prove that:

   \[
   \frac{a_1 + a_2 + \cdots + a_m}{m} \geq \frac{n + 1}{2}.
   \]

6. **Putnam 2003. A4.** Let $a, b, c, A, B, C$ be real, $a, A$ non-zero such that $|ax^2 + bx + c| \leq |Ax^2 + Bx + C|$ for all real $x$. Show that $|b^2 - 4ac| \leq |B^2 - 4AC|$.

7. **Putnam 2003. B6.** Show that

   \[
   \int_0^1 \int_0^1 |f(x) + f(y)| \, dx \, dy \geq \int_0^1 |f(x)| \, dx
   \]

   for any continuous real-valued function $f$ on $[0, 1]$. 