quiz 1

math133, linear algebra and geometry
summer 2022

Justify all your claims rigorously.

1. (sets and functions) Recall : Given a function $f : A \to B$ and two subsets $E \subset A$, $F \subset B$, we defined

$$f(E) = \{f(x) \in B\ ;\ x \in E\} \quad \text{and} \quad f^{-1}(F) = \{x \in A\ ;\ f(x) \in F\}.$$

a. Give an example of a function $f : A \to B$ and of a subset $E \subset A$ such that $f^{-1}(f(E)) \neq E$.

b. Let $f : A \to B$ be a function and let $E_1, E_2 \subset A$ such that $E_1 \subset E_2$. Show that $f(E_1) \subset f(E_2)$.

2. (computations involving complex numbers) Recall : Given a complex number $z = a + bi$, we defined its norm to be $|z| = \sqrt{a^2 + b^2}$.

a. Find a complex number $z \in \mathbb{C}$ satisfying the following equation :

$$(1 + i)z + 3 - 4i = -3 + 6i.$$

b. Let

$$z = \frac{1}{2} + \frac{\sqrt{3}}{2}i.$$

Show that $z^6 = 1$.

c. Is it true that $|z + w| = |z| + |w|$ for every complex numbers $z, w \in \mathbb{C}$? If it is, give a proof. If it is not, give a counter example.

d. Show that any complex number $z \neq 0$ can be written as $z = r \cdot \omega$ where $r \in \mathbb{R}$, $r > 0$ and $\omega \in \mathbb{C}$, $|\omega| = 1$.