quiz 2
math133, linear algebra and geometry
summer 2022

Justify all your claims rigorously.

1. (subspaces)

a. Consider $\mathbb{R}^4$ as an $\mathbb{R}$-vector space. Show that

$$\{(3x + y, x - y, 0, x) \in \mathbb{R}^4 \ ; \ x, y \in \mathbb{R}\}$$

is a subspace of $\mathbb{R}^4$.

b. Let $c \in \mathbb{R}$ and consider $\mathbb{R}^3$ as an $\mathbb{R}$-vector space. Show that

$$\{(x, y, z) \in \mathbb{R}^3 \ ; \ x + y + z = c\}$$

is a subspace of $\mathbb{R}^3$ if and only if $c = 0$.

2. (linear independence) Let $t \in \mathbb{R}$ and consider $u_1, u_2 \in \mathbb{R}^2$, where $u_1 = (t, 1)$ and $u_2 = (1, t)$. For which t are u_1 and u_2 linearly independent ?

3. (span) Let V be an $\mathbb{F}$-vector space where $\mathbb{F} = \mathbb{Q}$, $\mathbb{R}$ or $\mathbb{C}$. and let $u_1, u_2 \in V$. Show that

$$\mathrm{span}_{\mathbb{F}}\{u_1, u_2\} = \mathrm{span}_{\mathbb{F}}\{u_1 + u_2, u_1 - 2u_2\}.$$