# MATH133 – Summer 2022 – Tutorial 5

All questions denoted $(*)$ are not crucial to the course, but they give a taste of further topics in mathematics. I strongly encourage you to attempt all problems in this tutorial, but do not worry if you get stuck!

1. *Bases.* Find a basis for the following $\mathbb{F}$-vector spaces or subspaces
   (a) $\mathbb{F}$
   (b) $\{(x, y, z, w) \in \mathbb{F}^4 : x - y - 2z = 0\} \subseteq \mathbb{F}^4$
   (c) $\{(x, y, z) \in \mathbb{F}^3 : (x + y + z, x - y - z) = (0, 0)\} \subseteq \mathbb{F}^3$
   (d) $\{(x, y, z) \in \mathbb{F}^3 : (x + y + z, -x - y - z) = (0, 0)\} \subseteq \mathbb{F}^3$

2. *Lines and Planes.* Let $\mathbb{F} = \mathbb{R}$. For (c),(d) in the previous question, conclude whether or not the subspaces form a line or a plane.

3. *Intersections.* Do the following intersect?
   (a) The lines $\{t(-3, 5, 1) + (1, 2, 1) : t \in \mathbb{R}\}$ and $\{t(1, -4, -1) + (-1, 3, 1) : t \in \mathbb{R}\}$.
   (b) The lines $\{t(6, 5, -6)+(1, 1, 1) : t \in \mathbb{R}\}$ and $\{t(2, 1, -2)+(3, 0, 0) : t \in \mathbb{R}\}$.
   (c) The line $\{t(0, 1, 0) : t \in \mathbb{R}\}$ and the plane $x = 1$
   (d) The planes $x + y + 2z = 0$ and $x + y + 2z = 3$.

4. *Projections.* Given $v, w \in \mathbb{R}^n$, find $v_1, v_2 \in \mathbb{R}^n$ such that $v = v_1 + v_2$, where $v_1$ is parallel to $w$ and $v_2$ is perpendicular to $w$. We write $v_1 = \text{proj}_w(v)$. Find a gemoetric interpretation of $v_1$ in $\mathbb{R}^2$. *Hint: try* $v_1 = \frac{v \cdot w}{||w||} \cdot \frac{w}{||w||}$

5. Compute $\text{proj}_v(u)$ for the following $u, v \in \mathbb{R}^n$
   (a) $n = 1, u = 1, v = 2$
   (b) $n = 2, u = (5, 6), v = (0, 1)$
   (c) $n = 2, u = (cos\theta, sin\theta), v = (cos(\theta + \pi/2), sin(\theta + \pi/2))$
   (d) $n = 3, u = (3, 5, 3), v = (2, 2, 2)$
   (e) $n = 5, u = (1, 1, 1, 0, 0), v = (1, 2, 0, 0, 0)$

6. *Integrals.* $(*)$ Let $V$ be a vector space of continuous real functions on an interval $[a, b]$. Show that $\langle f, g\rangle := \int_a^b f(x)g(x)dx$ defines a dot product on this space. *Note: if you have never seen integrals before, you can ignore this question. I included it to give you an idea of different ways a dot product can be defined.*