quiz 1

math133, linear algebra and geometry
summer 2023

Justify all your claims rigorously.

1. Consider the following matrices :

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{pmatrix}, \qquad B = \begin{pmatrix} 1 & -1 & -3 \\ -1 & 2 & 5 \\ 2 & -3 & -7 \end{pmatrix}, \qquad x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, \qquad b = \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix},$$

where $x_1, x_2, x_3 \in \mathbb{R}$.

a. Compute the products $A \cdot B$ and $B \cdot A$.

b. Find $x_1, x_2, x_3 \in \mathbb{R}$ such that $A \cdot x = b$.

2. Let $\mathbb{K} = \mathbb{R}$ or $\mathbb{Q}$ and consider $A \in \mathbf{M}_{2\times 2}(\mathbb{K})$ such that

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}.$$

Moreover, suppose that $a_{11}a_{22} - a_{12}a_{21} \neq 0$. Show that the matrix

$$\begin{pmatrix} \frac{a_{22}}{a_{11}a_{22}-a_{12}a_{21}} & \frac{-a_{12}}{a_{11}a_{22}-a_{12}a_{21}} \\ \frac{-a_{21}}{a_{11}a_{22}-a_{12}a_{21}} & \frac{a_{11}}{a_{11}a_{22}-a_{12}a_{21}} \end{pmatrix} \in \mathbf{M}_{2\times 2}(\mathbb{K})$$

is the multiplicative inverse of the matrix A.

3. For $\theta \in \mathbb{R}$, define

$$K(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}.$$

Show that $K(\theta_1) \cdot K(\theta_2) = K(\theta_1 + \theta_2)$.

Cheatsheat. Recall the following trigonometric identities :

$$\sin(\theta_1 + \theta_2) = \sin(\theta_1)\cos(\theta_2) + \sin(\theta_2)\cos(\theta_1) \qquad \cos(\theta_1 + \theta_2) = \cos(\theta_1)\cos(\theta_2) - \sin(\theta_1)\sin(\theta_2)$$