quiz 2

math133, linear algebra and geometry
summer 2023

Justify all your claims rigorously.

1. Consider the following matrix :

$$A = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 0 & 2 \\ 1 & 0 & 3 \end{pmatrix} \in \mathbf{M}_{3\times 3}(\mathbb{R}).$$

a. Find elementary matrices $E_1, E_2, \ldots, E_k$ such that

$$E_k E_{k-1} \ldots E_2 E_1 A$$

is in *row echelon form*.

b. Find $x_1, x_2, x_3 \in \mathbf{M}_{3\times 1}(\mathbb{R})$ solving the following systems of linear equations :

$$Ax_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad Ax_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad Ax_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

c. Consider the matrix given by

$$B = \begin{pmatrix} | & | & | \\ x_1 & x_2 & x_3 \\ | & | & | \end{pmatrix} \in \mathbf{M}_{3\times 3}(\mathbb{R}),$$

i.e. the columns of B are the 3×1 matrices x_1, x_2, x_3 found in **b** previously. Compute the products AB and BA.

2. Let $A \in \mathbf{M}_{m\times n}(\mathbb{K})$ and $b \in \mathbf{M}_{m\times 1}(\mathbb{K})$. Consider the two following systems of linear equations :

$$Ax = b \tag{1}$$
$$Ax = 0 \tag{2}$$

Show that if $x_p \in \mathbf{M}_{n\times 1}(\mathbb{K})$ is a solution to the system (1) and $x_h \in \mathbf{M}_{n\times 1}(\mathbb{K})$ is a solution to the system (2), then $x_p + x_h$ is a solution to the system (1).