quiz 3
math133, linear algebra and geometry
summer 2023

Justify all your claims rigorously. Allotted time is 1 hour.

1. Consider the following vectors of $\mathbb{R}^3$:

$$v_1 = (2, 1, 1), \quad v_2 = (2, 0, -1), \quad v_3 = (4, 1, 0), \quad v_4 = (1, 1, 1).$$

    a. Are v_1, v_2, v_3, v_4 linearly independent ? Justify your answer.

    b. Do v_1, v_2, v_3, v_4 span $\mathbb{R}^3$? Justify your answer.

    c. Is $\mathcal{B} = \{v_1, v_2, v_3, v_4\}$ a basis of $\mathbb{R}^3$? If so, justify your answer. If not, find a subset of $\mathcal{B}$ which forms a basis of $\mathbb{R}^3$ and prove that it does in fact form a basis.

2. Let U, V, W be $\mathbb{K}$-vector spaces and let $T : U \to V$ and $S : V \to W$ be linear maps. Show that the composition

$$S \circ T : U \to W$$

is a linear map.