MATH133 – Summer 2023 – Tutorial 2

Please attempt the following exercises before the next tutorial. I encourage you to try all of them, but no worries of you get stuck!

1. (a) Find the inverse of $\begin{pmatrix} 3 & 4 \\ 2 & 3 \end{pmatrix}$.

   (b) Find the inverse of $\begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix}$.

   (c) Find the inverse of $\begin{pmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

   (d) Find the inverse of $\begin{pmatrix} 3 & 4 & 0 & 0 \\ 2 & 3 & 0 & 0 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & 5 & 2 \end{pmatrix}$.

2. Consider $A = \begin{pmatrix} 3 & 4 \\ 2 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, C = \begin{pmatrix} a & 0 \\ 0 & 1 \end{pmatrix}$ with $a \neq 0$.

   (a) Compute BA and AB. What do you notice?

   (b) Compute CA and AC.

   (c) Find B^{-1} and C^{-1}.

   Now consider $E_1 = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}, E_2 = \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix}$.

   (a) Compute $E_1E_2E_1$.

   (b) Compute $E_1E_2E_1A$.

3. Let $A = \begin{pmatrix} 0 & 1 & 3 \\ 1 & -1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$, and let $E_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, E_2 = \begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{pmatrix}$, $E_3 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

   (a) Compute $E_3E_2E_1A$.

   (b) What can we say about $E_3E_2E_1$?

4. (a) Show that if the system $Ax = b$ has a solution and A is invertible, then the solution is unique.

   (b) Show that if the matrix A has an inverse, then it is unique.