MATH133 – Summer 2023 – Tutorial 3

Please attempt the following exercises before the next tutorial. I encourage you to try all of them, but no worries of you get stuck!

1. Consider the linear system

$$\begin{cases} x - y + 2z & = 4 \\ 3x - 2y + 9z & = 14 \\ 2x + 4y + az & = b \end{cases}$$

   Find real numbers a, b such that

   (a) The system has a unique solution.

   (b) The system has infinitely many solutions.

   (c) The system is inconsistent.

2. Find solutions, if they exist, for the following systems of equations using Gaussian Elimination

   (a) $\begin{cases} x + y - 2z + w = 0 \\ x - y - w = 0 \\ 2y - z + 3w = 0 \end{cases}$

   (b) $\begin{cases} x + y - 2z + w = 1 \\ x - y - w = 5 \\ 2y - z + 3w = 3 \end{cases}$

   (c) $\begin{cases} x + y - 2z = 0 \\ x - y = 0 \\ 2y - 2z = 0 \end{cases}$

3. Recall from quiz 1 the following matrix $R_\theta = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$. Note also that there exists an identification of points (a, b) in $\mathbb{R}^2$ and 2 by 1 matrices $\begin{pmatrix} a \\ b \end{pmatrix}$ via

$$(a, b) \in \mathbb{R}^2 \leftrightarrow \begin{pmatrix} a \\ b \end{pmatrix} \in \mathbf{M}_{2\times 1}(\mathbb{R}).$$

   (a) Find R_θ^{-1}.

   (b) Remember that as Pythagoras once taught us, the distance of a point (a, b) to the origin is $\sqrt{a^2 + b^2}$. What is the distance to the origin for the point given by $R_\theta \begin{pmatrix} a \\ b \end{pmatrix}$?

(c) Call ϕ the angle between the positive x-axis and the vector (a, b). What can you say about the angle of $R_\theta \begin{pmatrix} a \\ b \end{pmatrix}$?

(d) Draw $\mathbb{R}^2$ and explain what happens when we take $R_\theta \begin{pmatrix} a \\ b \end{pmatrix}$.