Recall the construction of the nine-point circle:

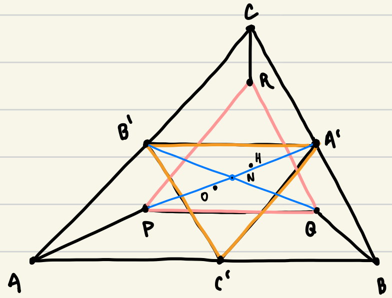
H : orthocenter of ABC

O : circumcenter of ABC

N : nine-point center of ABC

A'B'C' : medial triangle of ABC

PQR : P bisector of AH, Q of BH, R of CH.



We know that H is the circumcenter of A'B'C' and O is the orthocenter of A'B'C'. We also see that H is the orthocenter of PQR. We saw in the proof of the nine-pt circle that A'B'PQ is a rectangle. Same goes for B'C'QR and A'C'PR. Thus, $|A'B'| = |PQ|$, $|B'C'| = |QR|$ and $|A'C'| = |PR|$. By S-S-S, A'B'C' is congruent to PQR. Also, the distance from N to PQ is the same as N to A'B'. A rotation of 180° of A'B'C' around N gives PQR. This rotation interchanges O and H. Thus, $|ON| = |HN|$.