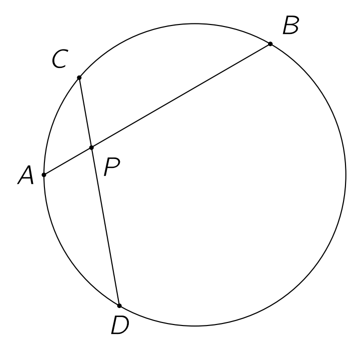
midterm 2
math228, classical geometry
fall 2021

The midterm 2 is worth 25 points. Justify all your claims rigourously.

less elementary geometry (12.5 points)

In this section, choose one question to answer.

1. Consider a circle of radius R and center O. Let A, B, C, D be points lying on the circle such that the chords AB and CD intersect at a point P inside the given circle.

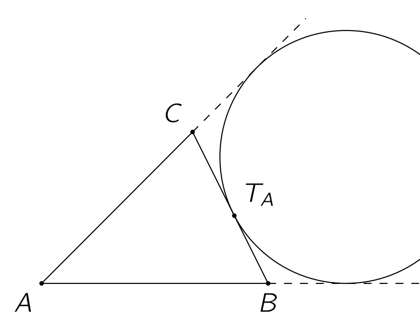


- **a.** Show that the triangles ADP and BCP are similar.
- **b.** Show that $|AP| \cdot |BP| = |CP| \cdot |DP|$.
- **c.** Conclude that

$$|AP| \cdot |BP| = R^2 - |OP|^2$$

by drawing a segment (diameter) passing through the point P and O.

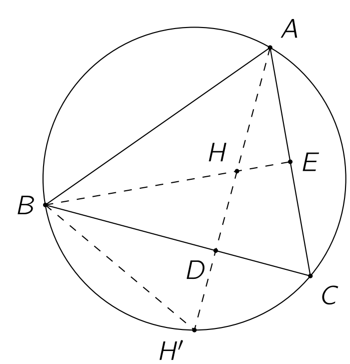
2. Let ABC be a triangle and consider the excircle tangent to BC and to the (extended) sides AB and AC. Let T_A the point of tangency of this excircle with BC. Show that with respect to A, T_A bisects the perimeter of ABC.



centers of triangles and Ceva's theorem (12.5 points)

In this section, choose one question to answer.

3. Consider an acute triangle ABC and let H be its orthocenter. Denote by D, E the feet of the altitude at A and B respectively and let $H' \neq A$ be the intersection of AD with the circumcircle of ABC.



- **a.** Show that $\angle CAD = \angle CBE$.
- **b.** Show that $\angle CBH' = \angle CAH'$.
- **c.** Using **a** and **b**, show that BD bisects the angle $\angle HBH'$.
- **d.** Conclude that $|DH| = |DH'|$ (in other words, the reflection of the orthocenter with respect to any side of ABC lies on the circumcircle).

4. Let ABC be a triangle and let X, Y, Z be points on BC, AC and AB respectively such that the cevians AX, BY, CZ are concurrent. Using Ceva's theorem, show that XY is parallel to AB if and only if Z is the midpoint of AB.
Suggestion : Study the triangles CXY and ABC.