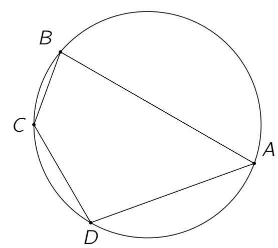
practice final
math228, classical geometry
fall 2021

Justify all your claims rigorously.

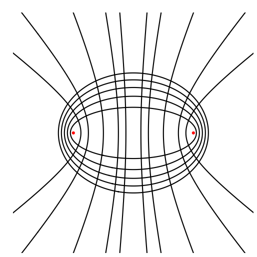


1. Let $ABCD$ be a cyclic quadrilateral. Prove that

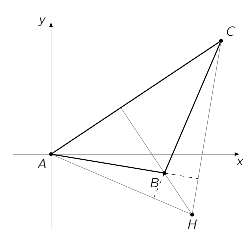
$$|AC| \cdot |BD| = |AB| \cdot |CD| + |AD| \cdot |BC|.$$

2. Given a triangle, show that the circumcircle of its orthic and of its medial triangles coincide.

3. Show that if ℓ is a line in an incidence geometry having a relation of betweeness, then there are infinitely many distinct points on ℓ.



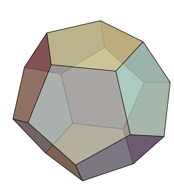
4. Show that if ϵ is an ellipse and υ is a hyperbola both having foci F_1, F_2, then ϵ and υ intersect at right angles.



5. Given three noncollinear points

$$A = (0, 0), \quad B = (B_x, B_y), \quad C = (C_x, C_y),$$

compute the coordinates of the orthocenter $H = (H_x, H_y)$ of the triangle ABC.
If you're feeling slightly more adventurous, you can also compute the coordinates of the circumcenter of ABC. Then, you can use it to compute the equation of the Euler line of ABC.



6. Draw the planar graph associated to the dodecahedron.