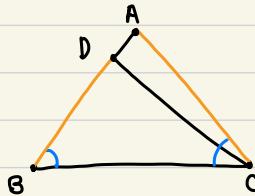
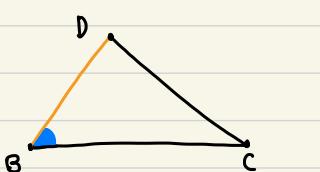


Prop 6, Book I. If a triangle has two angles equal to one another, then the sides subtending the equal angles will also be equal to one another. (AKA isosceles  $\Rightarrow$  isosceles).

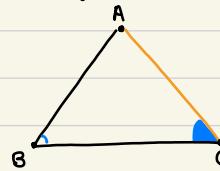
Proof. Let  $ABC$  be a triangle such that  $\angle ABC = \angle ACB$ , but  $|AB| > |AC|$ .



Pick  $D$  on  $AB$  such that  $|BD| = |AC|$  and draw segment  $DC$ . By SAS, we have



is congruent to



(they share the same base  $BC$ ). Thus,  $\angle ABC = \angle BCD$ . But,

$$\angle BCD + \angle ACD = \angle ACB$$

which implies

$$\angle ABC + \underbrace{\angle ACD}_{>0} = \angle ACB \stackrel{\text{hypothesis}}{=} \angle ABC \Rightarrow \angle ABC < \angle ACB$$

arriving at a contradiction.