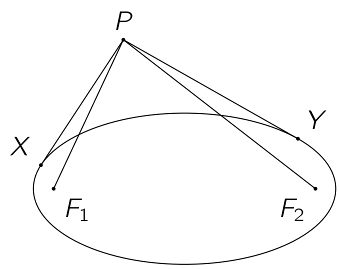
quiz 4
math228, classical geometry
fall 2021

The quiz is worth 10 points. Justify all your claims rigourously.

(10 points) **1.** Consider an ellipse ε with foci F_1, F_2. Let P be a point outside the ellipse and let X, Y be points on ε such that PX and PY are tangent to ε.



Let F_1' and F_2' be the reflections of F_1 and F_2 through PX and PY respectively.

a. Show that F_1' lies on F_2X and that F_2' lies on F_1Y.

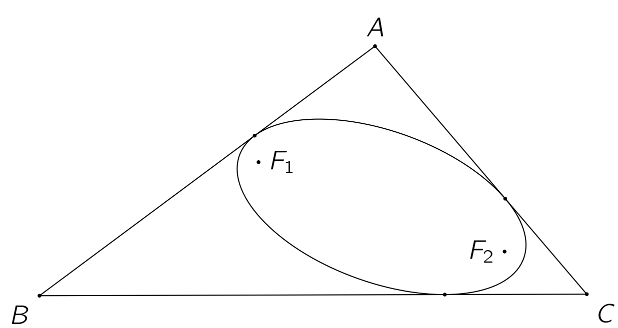
b. Show that the triangles $PF_1'F_2$ and PF_1F_2' are congruent.

c. Conclude that $\angle F_1PX = \angle F_2PY$.

(5 points) bonus **2.** First, we recall the definition of the isogonal conjugate of a point.

> Consider a triangle ABC and cevians AX, BY, CZ. Let J be the point of BC such that AJ is the angle bisector of $\angle BAC$. Let $\dot{X}$ be the point of BC such that $\dot{X} \neq X$ and $\angle JAX = \angle JA\dot{X}$. Similiarly, one defines $\dot{Y}$ and $\dot{Z}$. One can show that if AX, BY, CZ are concurrent, then so are $A\dot{X}, B\dot{Y}, C\dot{Z}$. Given a point P not on ABC, P defines a set of concurrent cevians AX, BY, CZ, where the intersection of AX, BY, CZ is P. The *isogonal conjugate* of P is defined to be the point $\dot{P}$ such that $\dot{P}$ is the intersection of $A\dot{X}, B\dot{Y}, C\dot{Z}$ (as defined above).

An *inellipse* of ABC is an ellipse tangent to AB, AC and BC.



Using **1c**, show that the foci F_1 and F_2 of an inellipse are isogonal conjugates.