

quiz 5

math228, classical geometry

fall 2021

The quiz is worth 10 points. Justify all your claims rigorously.

Recall the following axioms for points, lines and a relation of betweenness " $\star$ " :

- I1. For any two distinct points A, B , there exists a unique line ℓ containing A and B .
- I2. Every line contains at least two points.
- I3. There exists three noncollinear points.
- B1. If $A \star B \star C$, then A, B, C are three distinct points on line and $C \star B \star A$.
- B2. For any two distinct points A, B , there exists a point C such that $A \star B \star C$.
- B3. Given three distinct points on a line, one and only one of them is between the other two.
- B4. Let A, B, C be three noncollinear points and let ℓ be a line not containing any of A, B, C . If ℓ contains a point D lying between A and B , then it must also contain a point lying between A and C or a point lying between B and C but not both.

Consider the following set $\mathbf{X}$ and a set of its subsets $\mathbf{L}$ given by

$$\mathbf{X} = \{A, B, C, D, E, F\},$$

$$\mathbf{L} = \left\{ \{A, B\}, \{B, C\}, \{C, D\}, \{D, E\}, \{E, A\}, \{A, F, C\}, \{A, F, D\}, \{B, F, D\}, \{B, F, E\}, \{C, F, E\} \right\}.$$

Elements of $\mathbf{X}$ will be referred to as points and elements of $\mathbf{L}$ as lines. Suppose also that $\star$ is a relation between triplets of points such that

$$\begin{aligned} A \star F \star C, \quad C \star F \star A, \quad A \star F \star D, \quad D \star F \star A, \quad B \star F \star D, \quad D \star F \star B \\ B \star F \star E, \quad E \star F \star B, \quad C \star F \star E, \quad E \star F \star C \end{aligned}$$

hold true and are the only relations holding true.

(5 points) 1. For the previously defined sets of points and lines, show that the axioms I2, I3 are true and that I1 is false.

2. For the previously defined sets of points and lines and the relation " $\star$ " :

(5 points) a. Show that the axioms B1, B3 are true and that B2 is false.

(5 points) b. Does the axiom B4 hold? Justify your answer.

bonus