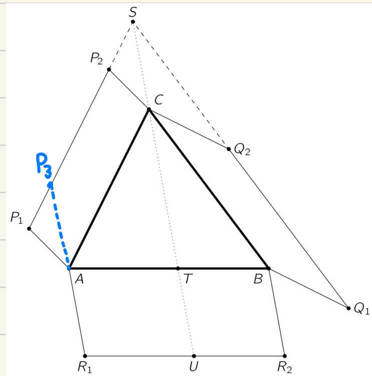
QUIZ 1



<u>Solution</u> a. We show that area(AP_1P_2C) = area(AR_1UT). The same proof can be applied to show that area(BQ_1Q_2C) = area(BR_2UT). Summing those two equations imply the desired result. First, extend AR_1 parallel to SU and write P_3 for the intersection point with P_1P_2 (see figure). Now, AP_3SC is a parallelogram by construction. It has the same base and same height as the parallelogram AP_1P_2C. We showed in class this implies

$$\text{area}(AP_1P_2C) = \text{area}(AP_3SC). \qquad (1)$$

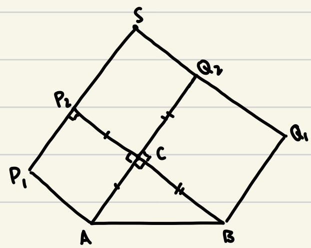
Now, the parallelogram AR_1UT has base TU which, by construction, is equal in length to SC. Since AP_3SC is contained in the same parallels as AR_1UT, the same result as before shows

$$\text{area}(AP_3SC) = \text{area}(AR_1UT). \qquad (2)$$

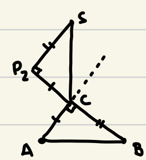
Combining (1) and (2), we get area(AP_1P_2C) = area(AR_1UT). This finishes the proof.

b. Suppose that $\angle ACB = \llcorner$ and that AP_1P_2C, BQ_1Q_2C are squares. By construction, we have that

$$\angle P_2CQ_2 = 4\llcorner - (\angle ACP_2 + \angle ACB + \angle BCQ_2) = \llcorner .$$



And, CP_2SQ_2 is a parallelogram, meaning $|CQ_2| = |P_2S|$ by a result from Euclid (parallelograms have opposite sides of = length). Lastly, $\angle CP_2S = 2\llcorner - \angle CP_2P_1 = \llcorner$. Now, by SAS, CP_2S is congruent to ABC and $|CS| = |AB|$. Consequently, the parallelogram AR_1R_2B will have four equal sides of length $|AB|$. Now, the segment CS (when extended) makes an angle with AB equal to $\angle BAC + \angle Q_2CS$. But, same process as before shows CSQ_2 is congruent to ABC. Thus, $\angle BAC + \angle Q_2CS = \angle BAC + \angle ABC = \llcorner$.



This implies SU is perpendicular to AB. Thus, AR_1R_2B is a square. Applying part a gives the pythagorean theorem.