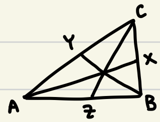
QUIZ 2

Solution. a. Let ABC be a triangle and let AX, BY, CZ be cevians of ABC. Then:

$$AX, BY, CZ \text{ are concurrent} \iff \frac{|AZ|}{|BZ|} \cdot \frac{|BX|}{|CX|} \cdot \frac{|CY|}{|AY|} = 1.$$



b. We show that X does not lie on AB. Same proof can be applied to show that X does not lie on AC. If X lies on AB, then,

$$s = \text{half the perimeter} = |AC| + |BC| + |BX| \overset{*}{=} |AX|$$

which implies

$$|AB| = |AX| + |BX| \overset{*}{>} |AX| \overset{*}{=} |AC| + |BC| + |BX| \overset{*}{>} |AC| + |BC|$$

where $*$ is true since $|BX| > 0$. But, the previous equation violates the triangle inequality (a contradiction).

c. If $s = \frac{1}{2}$ perimeter, we see that

$$|BZ| = |CY| = (s-a) \qquad |AZ| = |CX| = (s-b) \qquad |BX| = |AY| = (s-c)$$

by the def of X, Y, Z. Consequently,

$$\frac{|AZ|}{|BZ|} \cdot \frac{|BX|}{|CX|} \cdot \frac{|CY|}{|AY|} = \frac{(s-b)\cdot(s-c)\cdot(s-a)}{(s-a)\cdot(s-b)\cdot(s-c)} = 1 \quad \underset{\text{thm}}{\overset{\text{Ceva's}}{\Longrightarrow}} \quad AX, BY, CZ \text{ are concurrent.}$$