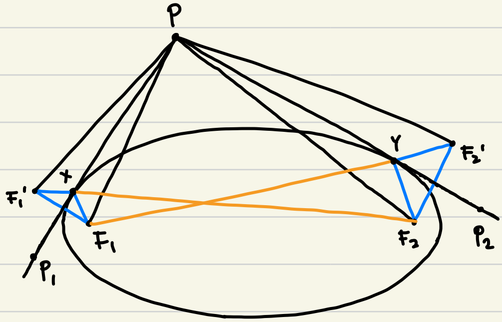
# QUIZ 4

1. a. By the properties of tangents to ellipses, we have that $\angle P_1XF_1 = \angle PXF_2$. But, since $F_1'$ is the reflection of $F_1$, $\angle P_1XF_1 = \angle P_1XF_1'$. But, $\angle P_1XF_1' = \angle PXF_2$ implies $F_1', X, F_2$ lie on a line. Same argument works for $F_2', Y, F_1$




b. Since $F_1'$ is the reflection of $F_1$, $|F_1X| = |F_1'X|$. Similarly, we have $|F_2Y| = |F_2'Y|$. Consequently,

$$|F_1'F_2| = |F_1'X| + |F_2X| = |F_1X| + |F_2X| \underset{X,Y \in E}{=} |F_1Y| + |F_2Y| = |F_1Y| + |F_2'Y| = |F_1F_2'|.$$

Again, by prop of reflections, $|PF_1| = |PF_1'|$ and $|PF_2| = |PF_2'|$. By SSS, $F_1F_2'P$ is congruent to $F_1'F_2P$.

c. By prop of reflections, $\angle F_1PX = \angle F_1'PX$ and $\angle F_2PY = \angle F_2'PY$. Thus, since $F_1F_2'P$, $F_1'F_2P$ are congruent,

$$\angle F_1'PF_2 = \angle F_1'PX + \angle F_1PX + \angle F_1PF_2 = \angle F_1PF_2 + 2\angle F_1PX$$
$$= \quad \Rightarrow \angle F_1PX = \angle F_2PY$$
$$\angle F_1PF_2' = \angle F_1PF_2 + \angle F_2PY + \angle F_2'PY = \angle F_1PF_2 + 2\angle F_2PY.$$

2. Let J be the intersection of BC with the angle bisector of $\angle BAC$. By 1c, we have $\angle F_1AB = \angle F_2AC$ since AB and AC are tangent to the ellipse. Consequently, since $\angle BAJ = \angle CAJ$, it follows that

$$\angle F_1AJ = \angle BAJ - \angle F_1AB = \angle CAJ - \angle F_2AC = \angle F_2AJ$$

Repeating the same argument for B and C shows that $F_1, F_2$ are isogonal conjugates.