MATH 347: FUNDAMENTAL MATHEMATICS, FALL 2015

HOMEWORK 1

Due on Wednesday, Sep 2

Exercises from the textbook. 1.13, 1.14, 1.15, 1.40, 1.45, 1.47(a), 1.51

Out-of-the-textbook exercises (these are as mandatory as the ones from the textbook)**.**

1. Prove that for any sets A and B that are subsets of an ambient set U, $(A \cup B)^c = A^c \cap B^c$, where the complements are taken within U. Draw the corresponding Venn diagram in your solution; it will guide you through your argument.

2. Let $\mathbb{R}^+$ denote the set of non-negative real numbers and let the function $f : \mathbb{R} \to \mathbb{R}^+$ be given by the formula $f(x) = \frac{x^2}{x^2+1}$. Determine the domain of f, its target set and its image. Find the simplest possible description of the image.

3. Let $f : X \to Y$. For $S \subseteq X$, let $f(S)$ denote the set $\{f(x) : x \in S\}$, in other words,

$$f(S) = \{y \in Y : \text{there is } x \in X \text{ with } f(x) = y\}.$$

Let $A, B \subseteq X$.

(a) Prove that $f(A \cup B) = f(A) \cup f(B)$.

(b) Prove that $f(A \cap B) \subseteq f(A) \cap f(B)$.

(c) Give an example of $f : X \to Y$ and sets $A, B \subseteq X$ that shows that the two sets in part (b) may not be equal (i.e. $f(A \cap B)$ can be a strict subset of $f(A) \cap f(B)$).

(d) Compare this problem with 1.51.

Exercises for fun, not for credit (don't have to turn in)**.**

From the textbook: 1.25, 1.47(b)