MATH 347: FUNDAMENTAL MATHEMATICS, FALL 2015

HOMEWORK 11

Due date: Dec 9 (Wed)

Exercises from the textbook. 14.5, 14.10, 14.43, 14.45, 14.58, 14.60

Out-of-the-textbook exercises (these are as mandatory as the ones from the textbook).

1. Let $c > 0$.

(a) Prove that the sequence $(\sqrt[n]{c})_n$ converges.

Hint: Consider the cases $c < 1$ and $c > 1$ separately.

(b) Find $\lim_{n\to\infty} \sqrt[n]{c}$.

Hint: Once you realize what the limit should be, suppose it isn't, so it is either bigger or smaller, and derive a contradiction in each case.

2. Let $(a_n)_n, (b_n)_n$ be sequences of positive reals and suppose that for each $n \in \mathbb{N}$, $b_n \geq a_n$. Prove that if $\sum_{n=1}^{\infty} a_n$ diverges, then so does $\sum_{n=1}^{\infty} b_n$.

3. (a) Prove that $\sum_{n=2}^{\infty} \frac{1}{(n-1)n}$ converges and find the sum.

(b) Conclude that $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges. You may not use the result proved in class that $\sum_{n=1}^{\infty} \frac{1}{n^{1+p}}$ converges for any $p > 0$.