Math 347H: Fundamental Math (H) HOMEWORK 4 **Due date: Oct 12 (Thu)**

Exercises from Sally's book. 1.6.2, 1.6.15, 1.6.16

Other (also mandatory) **exercises.**

1. Given a partition $\mathcal{C}$ of a set X, it was proven in class that the binary relation $E_{\mathcal{C}}$ on X is an equivalence relation. Prove this again on your own very carefully (without skipping any steps). Just in case, we recall the definition of $E_{\mathcal{C}}$: for any $x, y \in X$,
$$x E_{\mathcal{C}} y :\Leftrightarrow \exists A \in \mathcal{C} \text{ such that } x, y \in A.$$

2. Conversely, given an equivalence relation E on a set X, it was proven in class that the set of E-classes form a partition $\mathcal{C}$ of X and that $E_{\mathcal{C}}$ is exactly E. Prove this again on your own following the steps below.
   (a) Prove that for each $x, y \in X$, xEy if and only if $[x]_E = [y]_E$.
   (b) Prove that $\bigcup_{x \in X} [x]_E = X$.
   (c) Prove that for each $x, y \in X$, if $[x]_E \cap [y]_E \neq \emptyset$ then $[x]_E = [y]_E$.
   (d) Conclude that $E_{\mathcal{C}} = E$.

3. Let $f : \mathbb{Z} \to \mathbb{Z}$ be defined by $f(z) := z^2$. Define the relation E_f on $\mathbb{Z}$ by putting
$$x E_f y :\Leftrightarrow f(x) = f(y).$$
It was proven in class (for any function on any set) that this is an equivalence relation. Explicitly describe and list all E_f-equivalence classes.

4. Let $G := (V, E)$ be an undirected graph with no loops, i.e. V is the set of vertices and $E \subseteq V^2$ is the set of edges, which is irreflexive and symmetric. For vertices $x, y \in X$, we say that y is *adjacent* to x (or y is a *neighbor* of x) if there is an edge $(x, y) \in E$. Assuming that V is finite, the *degree* of each vertex $v \in V$, denoted by $\deg_G(v)$, is the number of neighbors of x. Prove that the sum of all degrees, i.e. $\sum_{v \in V} \deg_G(v)$, has to be an even number.

5. There was a party of 170 people in which every person shook some other people's hands (at most one hand per person). It is possible that a person didn't shake anyone's hand. Prove that there are two people that shook equal number of hands.

   HINT: Prove by contradiction. Use the statement of Question 4.