Math 347H: Fundamental Math (H) HOMEWORK 6 **Due date: Oct 26 (Thu)**

Exercises from Sally's book. 1.7.6, 1.7.22

Other (also mandatory) **exercises.**

1. Let $f : X \to Y$ be a function, $A, A_1, A_2 \subseteq X$, and $B, B_1, B_2 \subseteq Y$.

(a) Prove that $f^{-1}(B_1 \setminus B_2) = f^{-1}(B_1) \setminus f^{-1}(B_2)$. Deduce that $f^{-1}(B^c) = f^{-1}(B)^c$, where the complement on the left is within the set Y and the complement on the right is within the set X.

(b) Prove that $f(A_1 \setminus A_2) \supseteq f(A_1) \setminus f(A_2)$ and provide an example showing that the reverse inclusion need not hold.

(c) Show by an example (or examples) that, in general, neither of the inclusions hold between the sets $f(A^c)$ and $f(A)^c$.

(d) Determine which condition (injectivity/surjectivity) guarantees that $f(A^c) \subseteq f(A)^c$.

(e) Determine which condition (injectivity/surjectivity) guarantees that $f(A^c) \supseteq f(A)^c$.

2. Let $f : X \to Y$ and $g : Y \to Z$ be functions.

(a) Prove that if f and g are injective, then $g \circ f$ is also injective.

(b) Prove that if f and g are surjective, then $g \circ f$ is also surjective.

(c) Assuming that $g \circ f$ is injective, prove that f is injective, but provide an example showing that g may not be injective.

(d) Assuming that $g \circ f$ is surjective, prove that g is surjective, but provide an example showing that f may not be surjective.

(e) Prove that if $g \circ f$ is injective but g isn't injective, then f isn't surjective.[1]

(f) Prove that if $g \circ f$ is surjective but f isn't surjective, then g isn't injective.[1]

[1] Thanks to Kieran Kaempen for suggesting this question.