Math 347H: Fundamental Math (H) HOMEWORK 9 **Due date: Nov 30 (Thu)**

1. Do the following. Include the steps of your calculations.

(a) Write $(230102)_4$ in its decimal expansion.

(b) Write $(5.90625)_{10}$ in its binary expansion.

REMARK:We didn't really cover the algorithm for conversion of fractional expansions, so part of the challenge of this question is to come up with an algorithm first. It should be a natural extension of the algorithm for natural numbers.

(c) Write $(10459)_{10}$ in hexadecimal (16-ary) expansion using $0, 1, \ldots, 9, A, B, C, D, E, F$ as digits.

(d) Write the rational number $\frac{21}{8}$ in its decimal expansion.

(e) Write the rational number $\frac{19}{6}$ in its decimal expansion.

2. Recall the definition of the set of reals $\mathbb{R}$ from your lecture notes and prove directly (without using any other statement proven in class) that $(0,1)$ (and hence also $\mathbb{R}$ itself) is uncountable, using a direct diagonalization argument.

HINT: Supposing towards a contradiction that $\mathbb{R}$ is countable allows for listing the members of $(0,1)$ one below another, which results in a matrix of digits. Looking at the diagonal digits, create a real that is not on that list.

3. The goal of this question is to define addition of two nonnegative reals x, y. For example, $x := 215.69835741\ldots$ and $y := 6.50293294\ldots$. Write enough extra 0s in front of either x or y to make the number of digits before the . equal: $y = 006.50293294\ldots$. Furthermore, write an additional 0 in front of both of them and declare its position as the 0^{th} position:

$$
\begin{array}{cccccccccccccccc}
\text{positions} & : & 0 & 1 & 2 & 3 & & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & \ldots \\
 & & \downarrow & \downarrow & \downarrow & \downarrow & & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \\
x & = & 0 & 2 & 1 & 5 & . & 6 & 9 & 8 & 3 & 5 & 7 & 4 & 1 & \ldots \\
+ & & & & & & & & & & & & & & & \\
y & = & 0 & 0 & 0 & 6 & . & 5 & 0 & 2 & 9 & 3 & 2 & 9 & 4 & \ldots \\
:= & & & & & & & & & & & & & & & \\
z & = & z_0 & z_1 & z_2 & z_3 & . & z_4 & z_5 & z_6 & z_7 & z_8 & z_9 & z_{10} & z_{11} & \ldots
\end{array}
$$

For each position $i \in \mathbb{N}$, let x_i and y_i denote the digit in the i^{th} position of x and y, respectively. Let $\text{overflow}(x, y, i)$ denote the least position $j > i$ such that $x_j + y_j \geq 10$; if such a j does not exist, we let $\text{overflow}(x, y, i) := \infty$, declaring the symbol ∞ greater than any natural number. Furthermore, define $\text{carry}(x, y, i) := 1$ if for each $j \in \mathbb{N}$ with $i < j < \text{overflow}(x, y, i)$, $x_j + y_j = 9$; otherwise, define $\text{carry}(x, y, i) := 0$.

(a) For each position $i \in \mathbb{N}$, provide a definition (formula) of z_i using $\text{carry}(x, y, i)$.

(b) Calculate z for $x := 15.3066666666\ldots$ (continue with 6s) and $y := 390.3827355555\ldots$ (continue with 5s).

(c) Calculate z for $x := .4444444444\ldots$ (continue with 4s) and $y := .5555555555\ldots$ (continue with 5s).