Complete, last update: Apr 9 at 11:30am

Math 432: Set Theory and Topology HOMEWORK 10 Due date: Apr 13 (**Thu**)

Exercises from Kaplansky's book.

Sec 4.3: 10, 18

Sec 4.4: 1, 5, 8

1. Let $(x_n)_{n\in\mathbb{N}}, (y_n)_{n\in\mathbb{N}} \subseteq \mathbb{R}$ be sequences converging to $x, y \in \mathbb{R}$, respectively. Prove the following using the ε-N definition of limit.

(a) $x_n + y_n \to x + y$.

(b) $x_n \cdot y_n \to x \cdot y$.

(c) Assuming that $x \neq 0$, $\frac{1}{x_n} \to \frac{1}{x}$.

Remark. $x_n \to x \neq 0$ implies $\forall^\infty n \; x_n \neq 0$, so, up to throwing out the first finitely many elements, the sequence $\left(\frac{1}{x_n}\right)_{n\in\mathbb{N}}$ makes sense.

Caution. Make sure the δ you choose does not depend on n, x_n, or y_n. It can only depend on ε, x, and/or y.

2. Let $A \subseteq \mathbb{R}$ be bounded (with respect to either metric or order, these are equivalent for $\mathbb{R}$). Prove that there is a sequence $(a_n)_{n\in\mathbb{N}} \subseteq A$ converging to $\sup A$. Same is true for $\inf A$.

3. (Monotone Convergence Theorem) Let $(x_n)_{n\in\mathbb{N}} \subseteq \mathbb{R}$ be an increasing (i.e. nondecreasing) bounded sequence. Prove that $x_n \to \sup \{x_k : k \in \mathbb{N}\}$. Same is true with decreasing and inf.