How to Win Dots and Boxes

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Lessons and Play: M.H. Albert,R.J. Nowakowski, D. Wolfe

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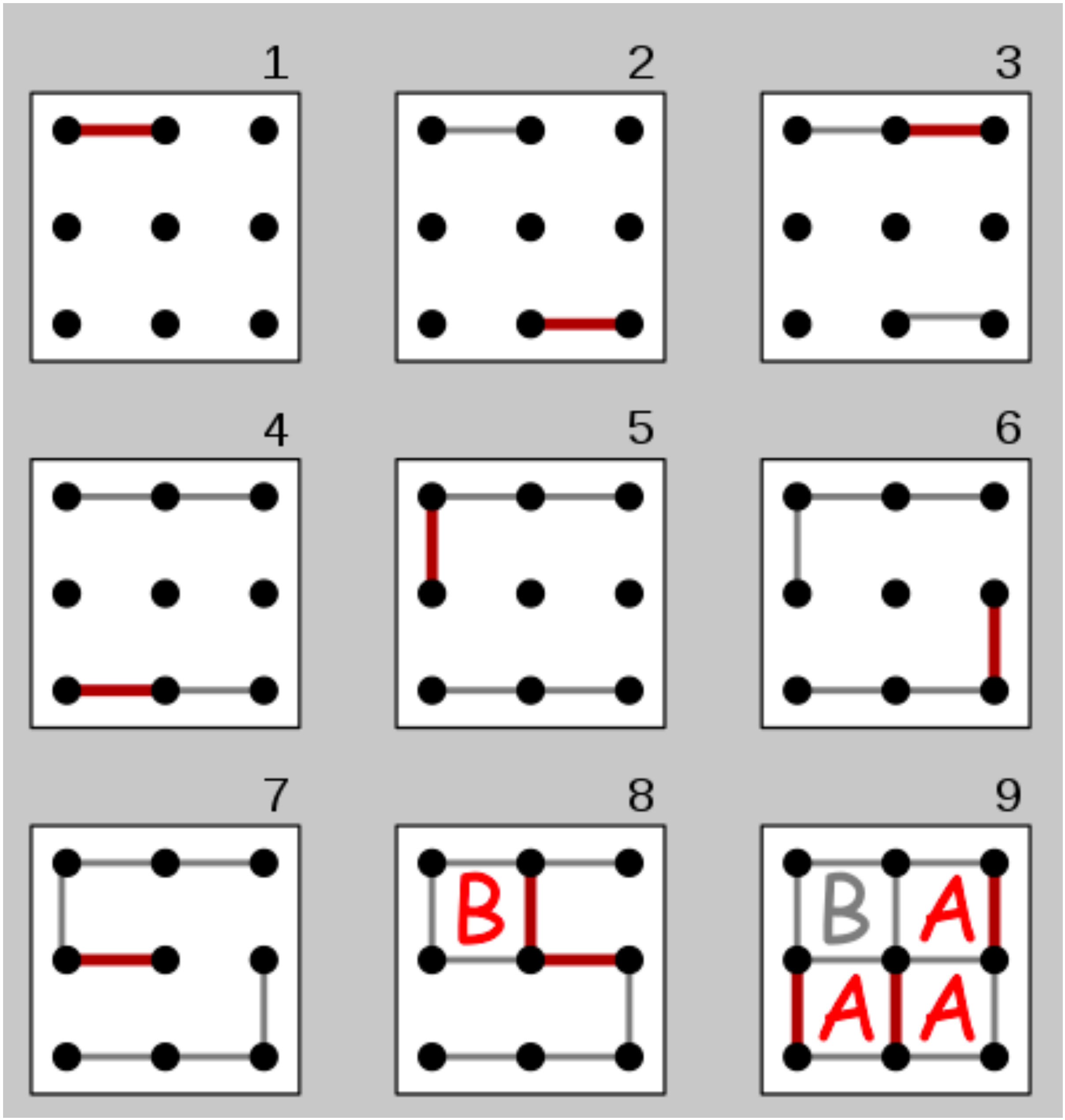
Rules

Dots and Boxes

- Two players, Alice and Bob, start from a (rectangular) array of vertices (dots) and take turns to add edges horizontally or vertically.
- The player who completes the fourth side of a unit square (box) earns one point and takes another turn.
- The game ends when there is no more box can be completed. Whoever has more boxes is the winner (optimization).
- Note: there are two phases for this game (1. connecting vertices 2. collecting boxes).



Cr: https://en.wikipedia.org/wiki/Dots_and_Boxes

Let's Play

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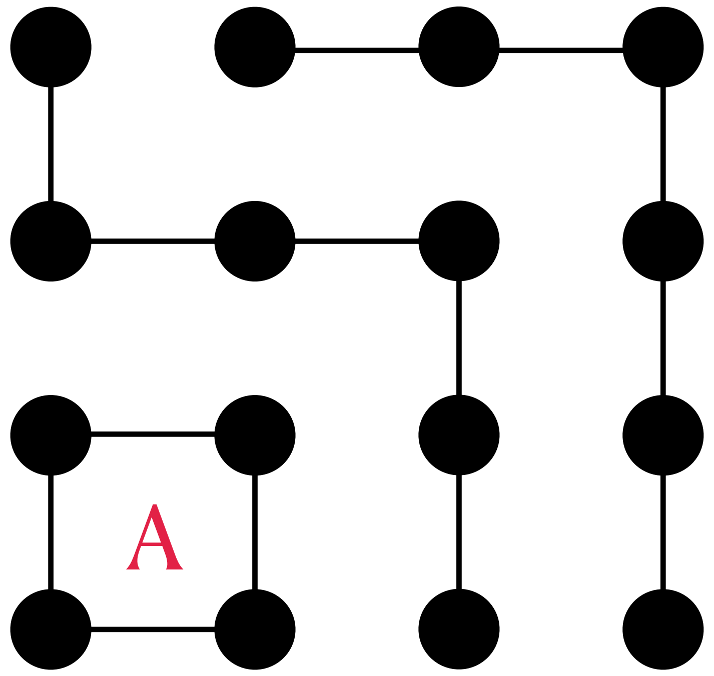

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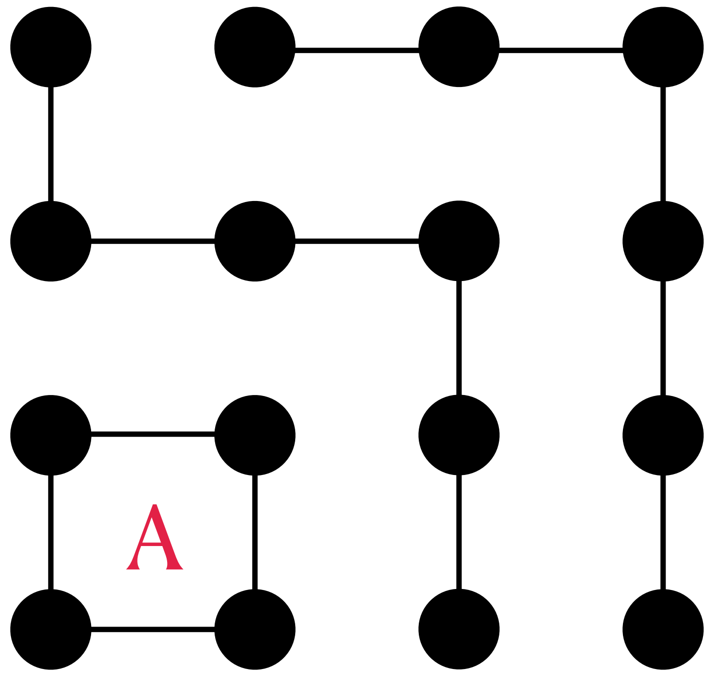

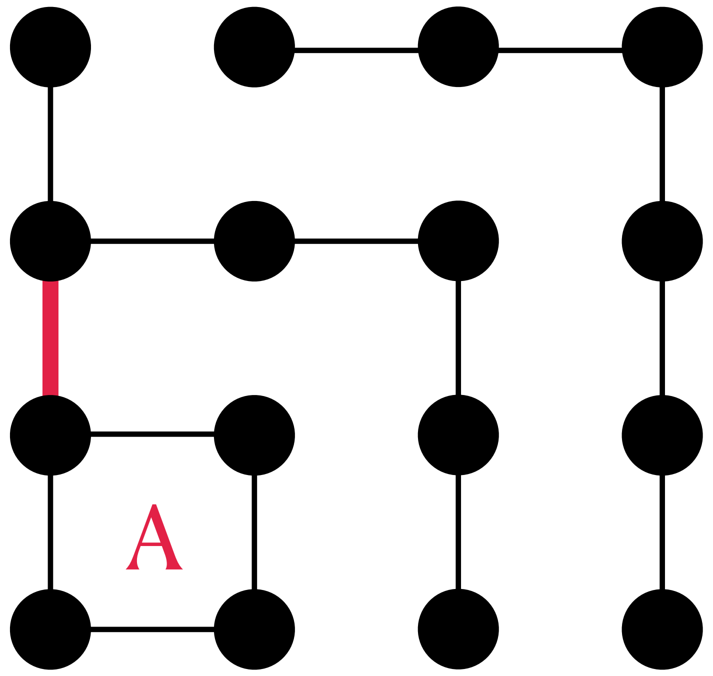

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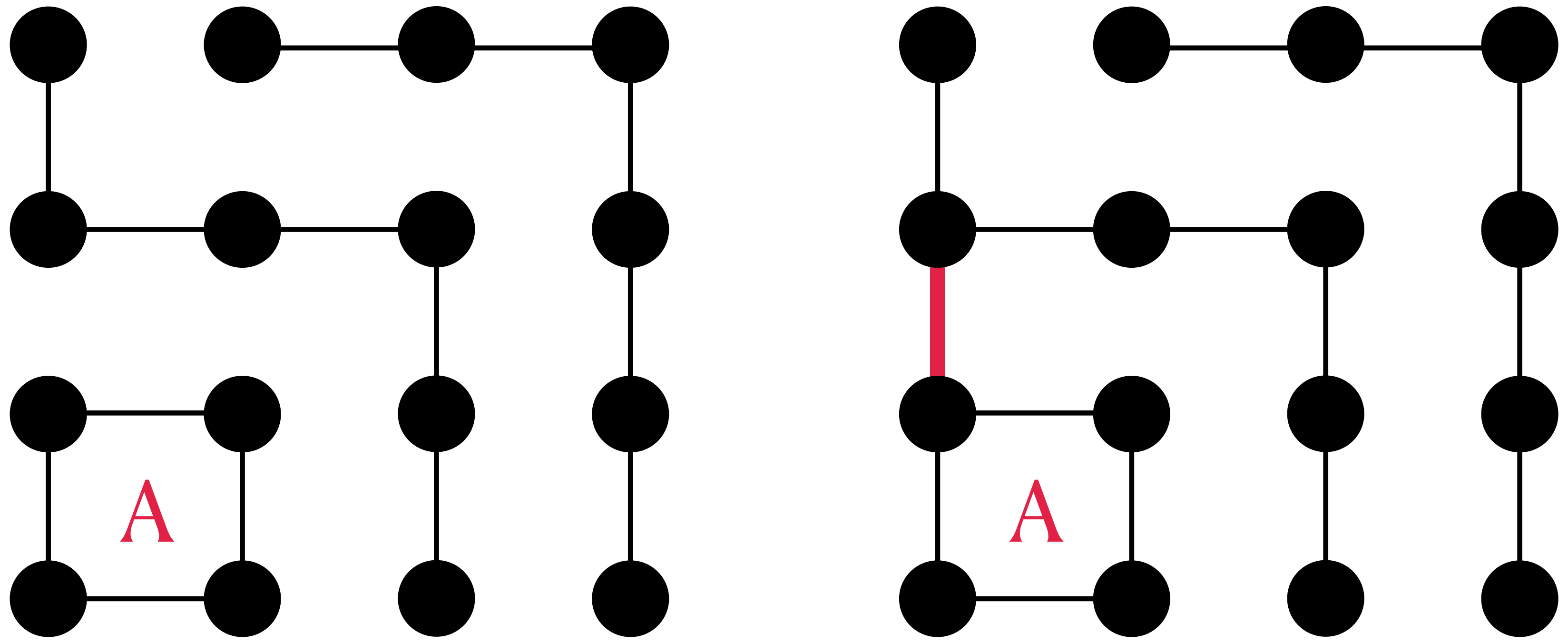

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- Suppose you are Bob, what is your next move?

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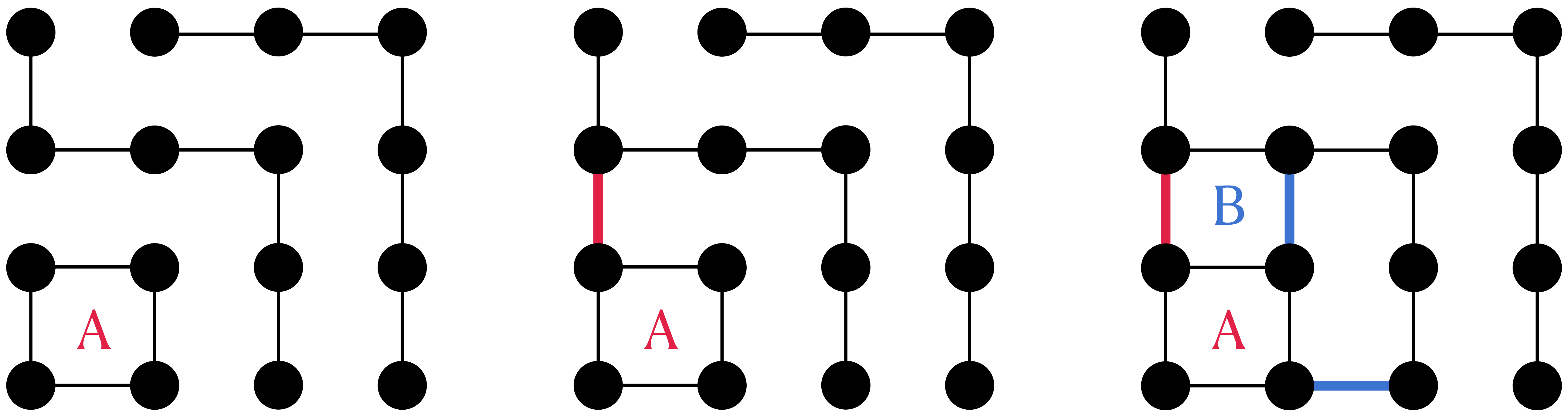

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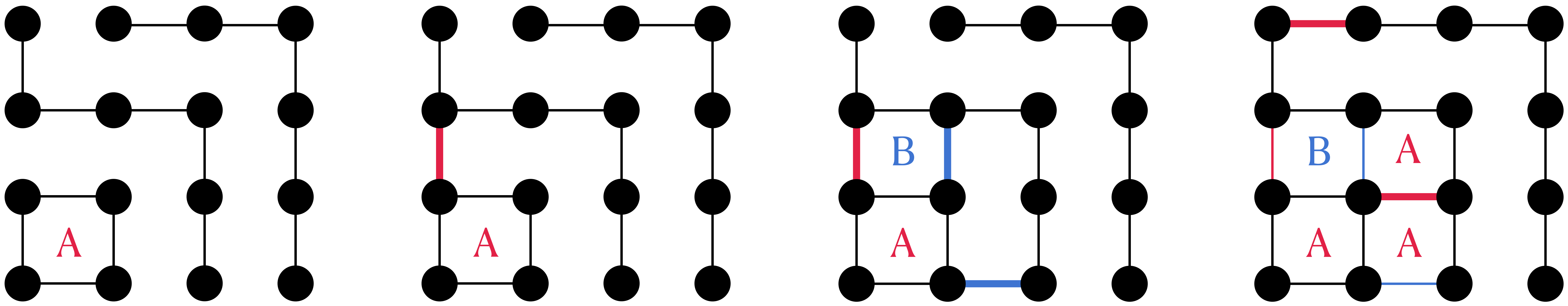

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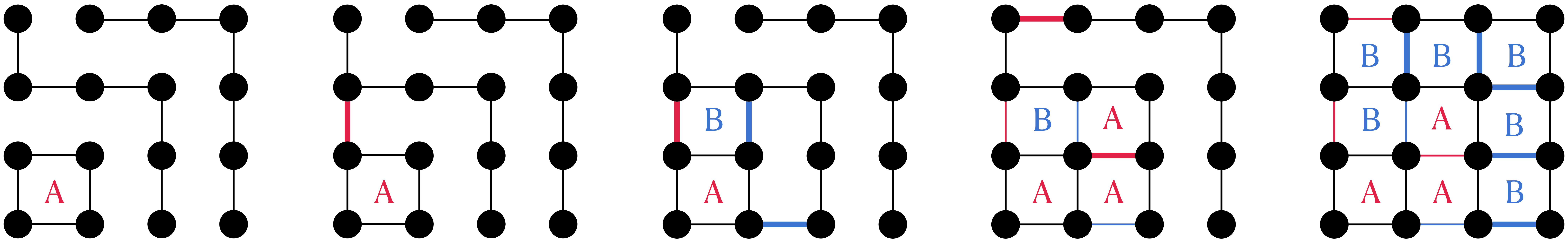

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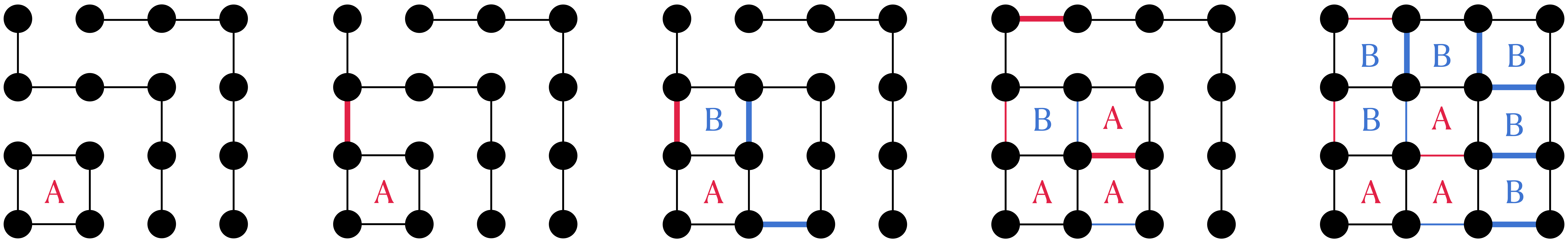

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- Suppose you are Bob, what is your next move?
    - Choice 1: greedily take the chain, and open the long chain to Alice

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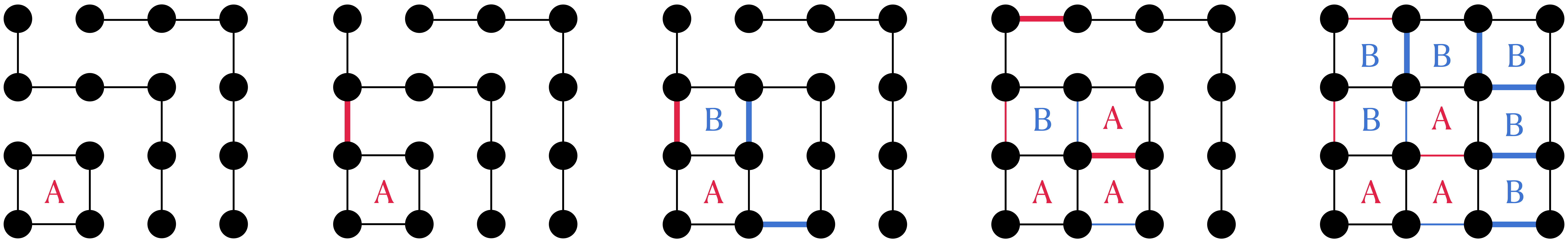


- It's Alice's turn, she want to force Bob to play on the shorter chain.
- Suppose you are Bob, what is your next move?
  - Choice 1: greedily take the chain, and open the long chain to Alice
  - Choice 2: sacrifice 2 boxes, Alice is forced to open the long chain and then Bob can take the long chain

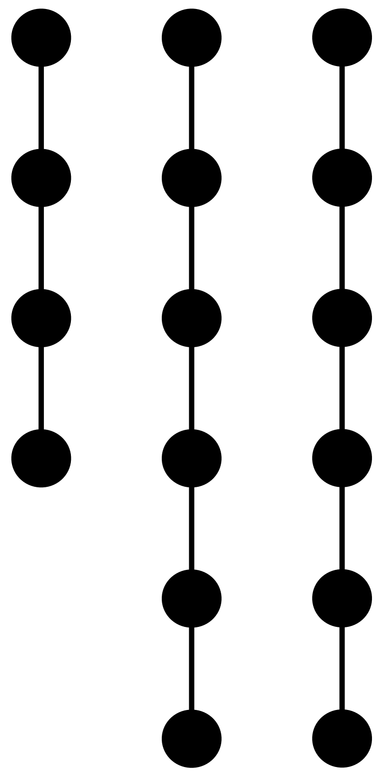
Double-Crossing

A Winning Strategy

- When Bob was forced to take a chain opened by Alice, he could close it with a double-cross move: sacrificing 2 boxes, but then Alice is forced to open a longer chain to Bob.
- After double-crossing, Bob gains control of the game. Otherwise, Alice has the control.
- The player who has control usually wins when there are several long chains.
- A long chain is a chain contains 3 or more boxes: it takes at least 3 boxes to complete a double-crossing move. (Note that a 2-box chain can be broken into 2 individual boxes, preventing the opponent from double-crossing.)

Long Chain

Long Chain



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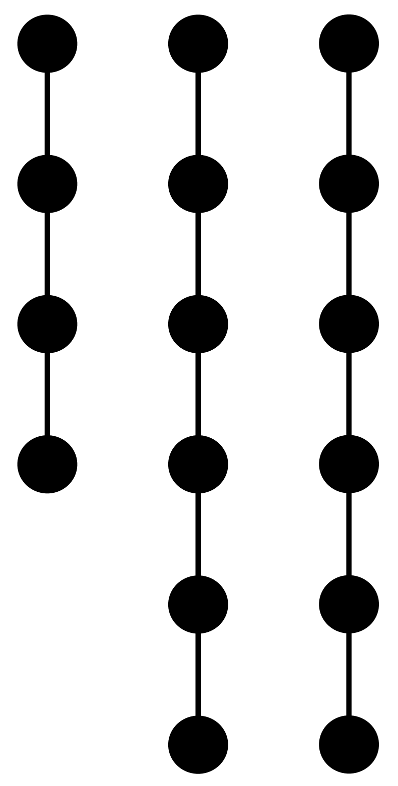

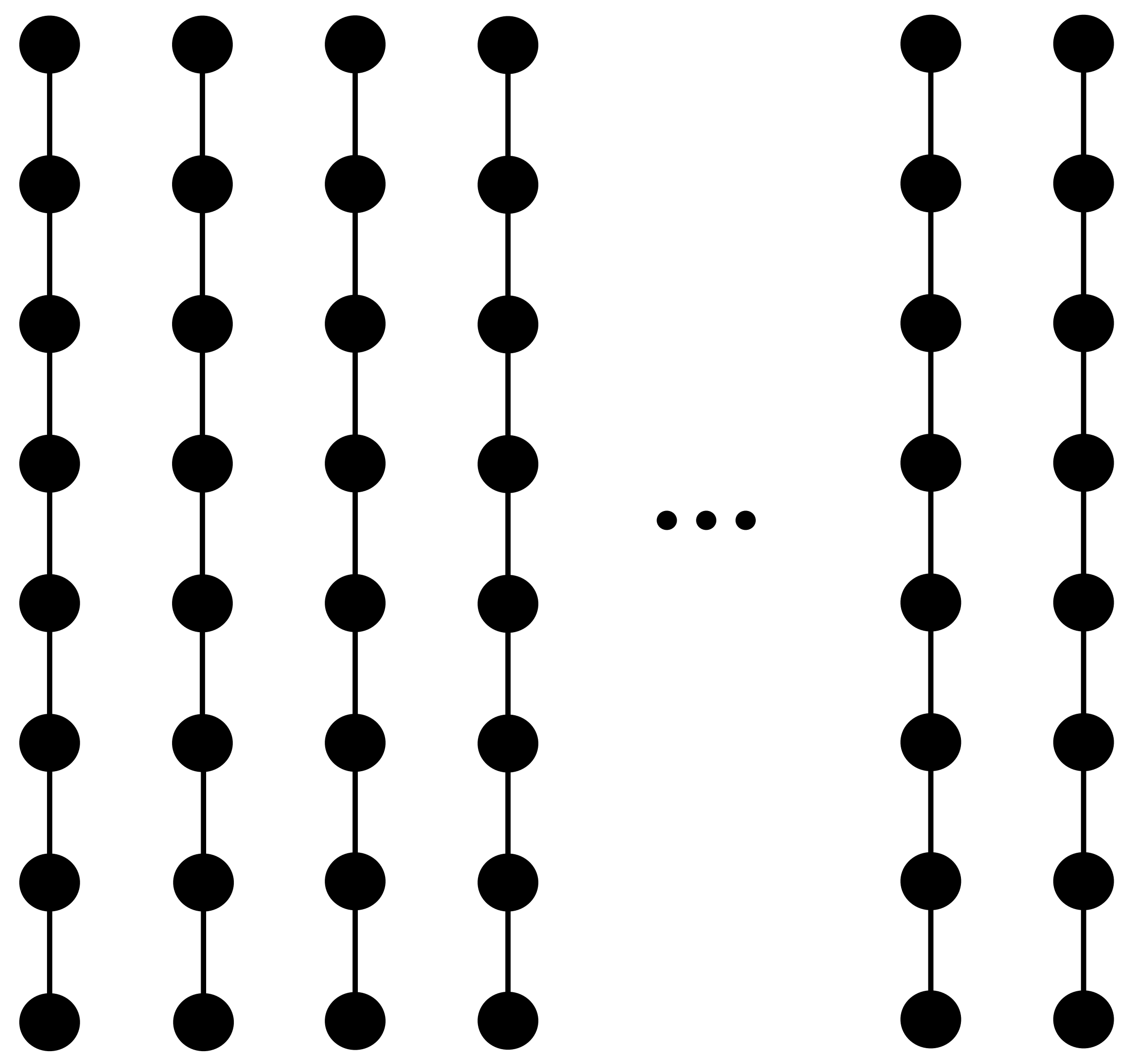

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- In general, if there are more than 1 long chains, it is always a winning strategy to take the control by double-crossing.

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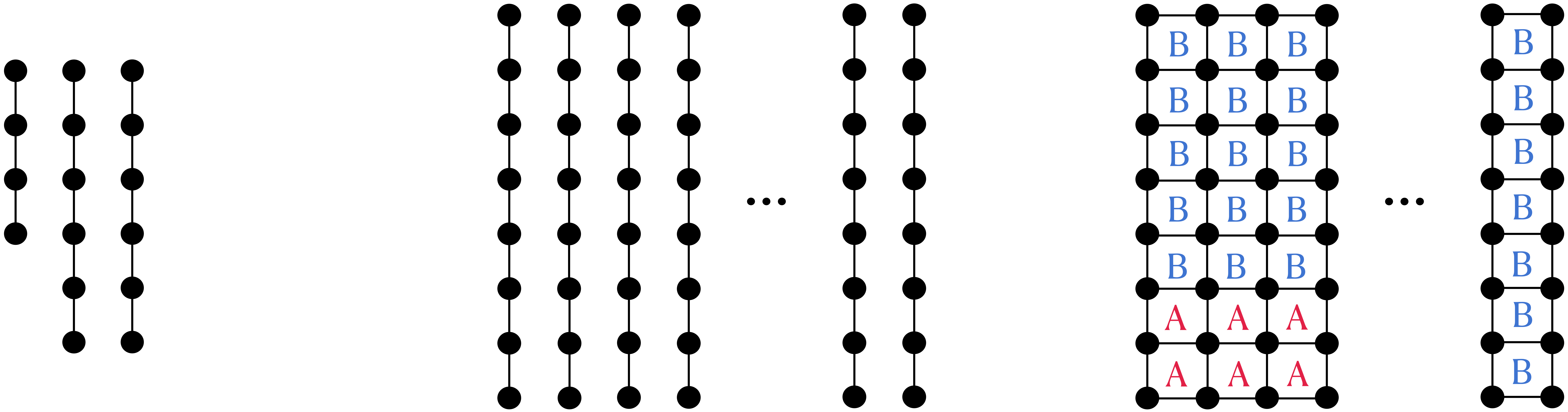


- In the previous game board, there were 2 long chains.
- In general, if there are more than 1 long chains, it is always a winning strategy to take the control by double-crossing.
- Let m be the number of long chains and n be the number of boxes, then if Bob use the strategy of double-crossing, he can score $n - 2m + 2 > 0$

Definitions

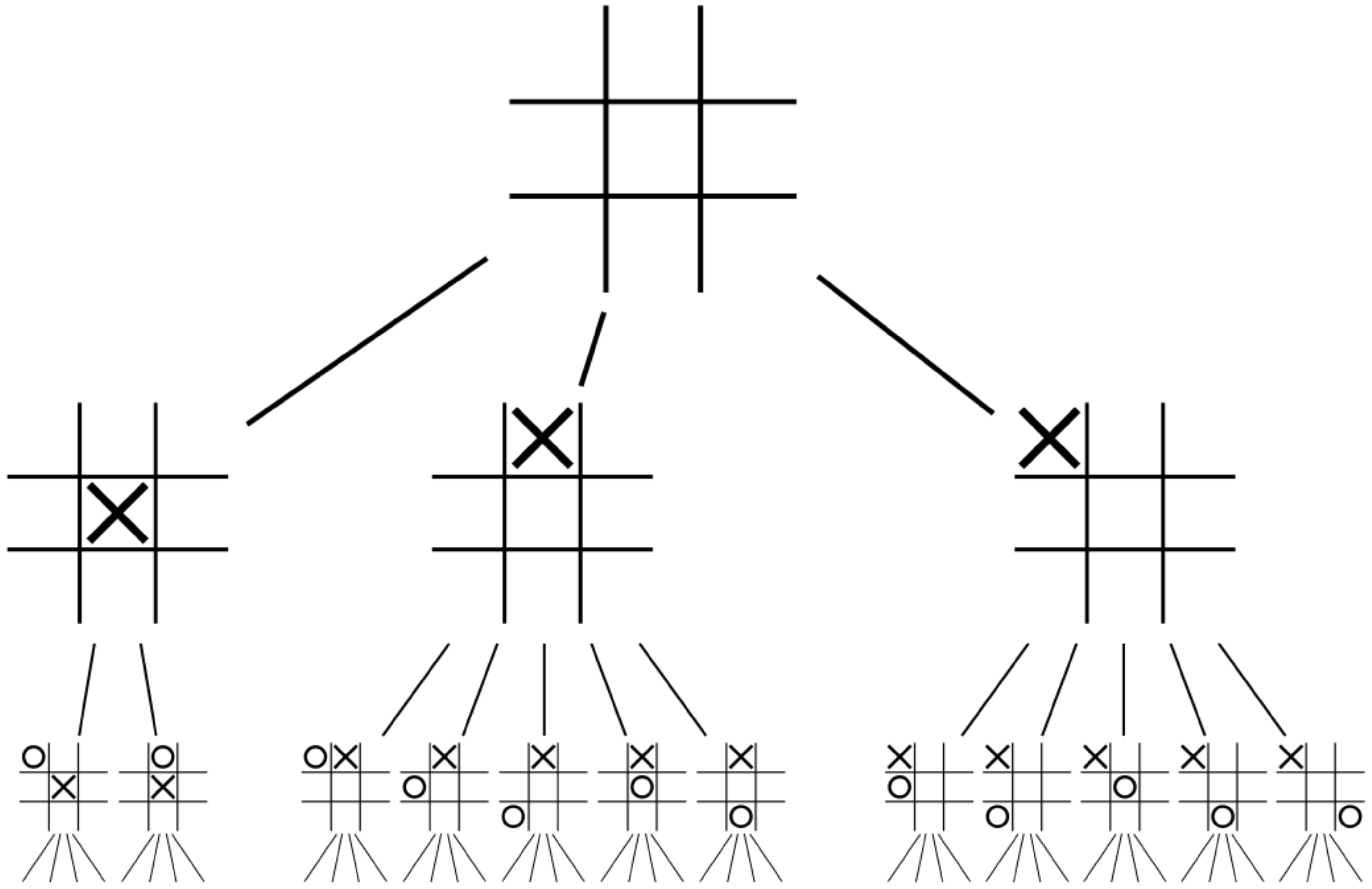
Combinatorial Game Theory

- *Combinatorial game theory* studies two-player sequential games: players move sequentially as opposed to simultaneously in economic game theory.
- The winners in most combinatorial games depend on the last player, in contrast to Dots and Boxes.
- By convention $\mathscr{L}$ and $\mathscr{R}$ are used for each of the two players (instead of Alice and Bob).
- A *game* G is defined by $G = \{\mathscr{G}^L \mid \mathscr{G}^R\}$ where $\mathscr{G}^L$ and $\mathscr{G}^R$ stand for the set of left and right options respectively.

Game Tree

Combinatorial Game Theory

- For impartial games, the set of options for *left* and *right* are the same.
- We can draw a game tree of a position:
  - The root node is the original position
  - Create a node for each option from the root and connect to the root
  - For each node create node for its options and connect to the node
  - Repeat until there is not more options



Cr: https://en.wikipedia.org/wiki/Game_tree

More Definitions

Birthday of a Game

- Recall $G = \{\mathscr{G}^L \mid \mathscr{G}^R\}$ where $\mathscr{G}^L$ and $\mathscr{G}^R$ stand for the set of left and right options.
- The *birthday* of a game $G = \{\mathscr{G}^L \mid \mathscr{G}^R\}$ is defined as 1 + max birthday of any game in $\mathscr{G}^L \cup \mathscr{G}^R$.
- Base case: if $\mathscr{G}^L = \mathscr{G}^R = \emptyset$, then the birthday of G is 0, i.e. $0 = \{\ \mid\ \}$.
- Apply the definition recursively we have:
    - $1 = \{0 \mid\ \}$
    - $-1 = \{\ \mid 0\}$
    - $* = \{0 \mid 0\}$

Thanks