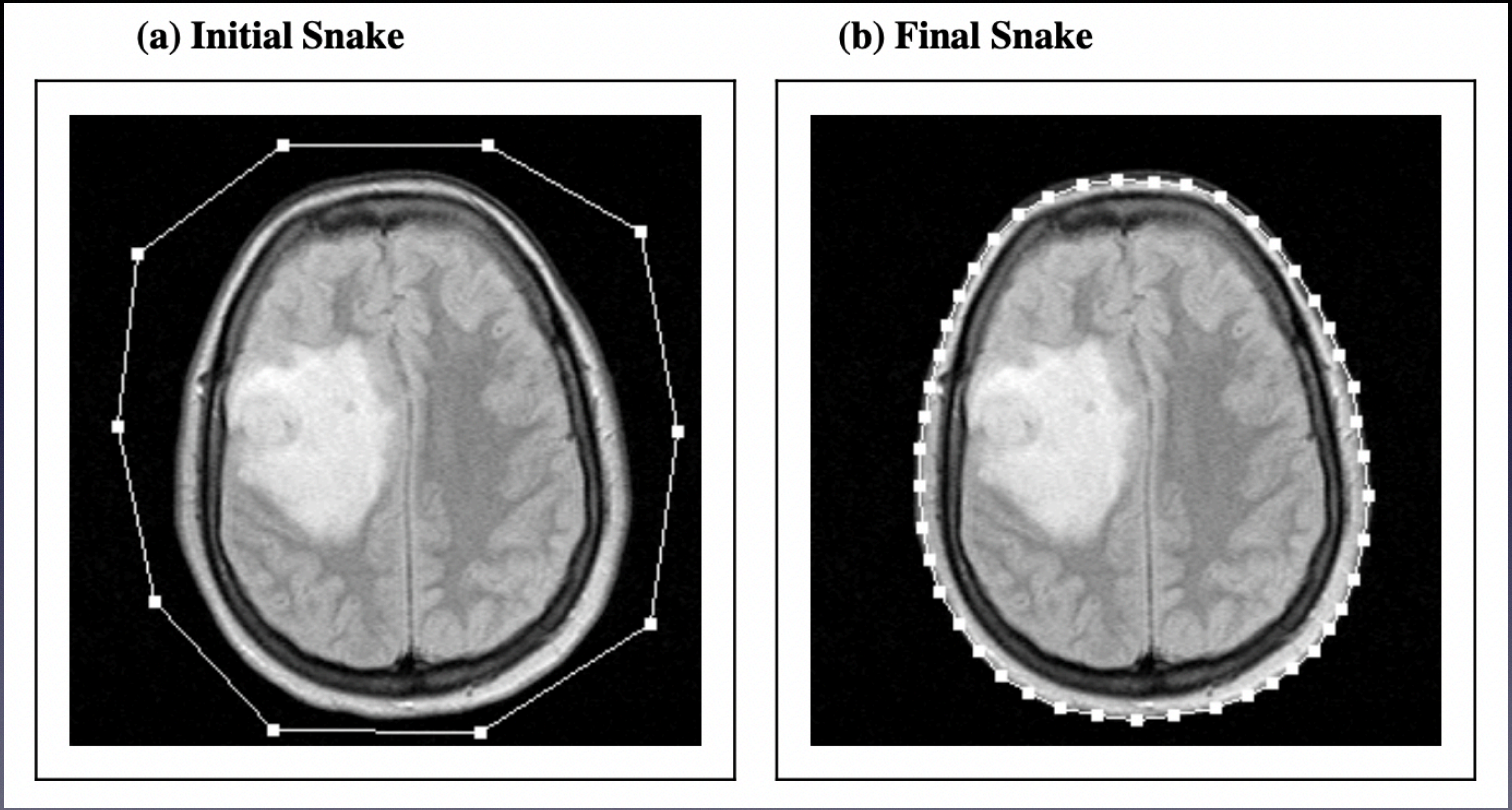
Active Contour

Snakes

- To match a deformable model to an image
- Energy minimization
- A low-level mechanism which seeks local minima


(a) Initial Snake
(b) Final Snake

Energy

- Internal energy of the spline
- External constraint
- Image forces
- $$E_{snake} = \int_A^B E_{int}(\gamma(s)) + E_{image}(\gamma(s)) + E_{ext}(\gamma(s))ds$$

Internal Energy

- Internal energy - tension and stiffness
- $E_{int} = \alpha||\gamma'(s)||^2 + \beta||\gamma''(s)||^2$
- First-order term - contract like an elastic band
- Second-order term - resist bending

External Energy

- External energy
- Spring - attraction $E_{ext}(\mathrm{x}) = k|\mathrm{i} - \mathrm{x}|^2$
- Volcano - repulsion $E_{ext}(\mathrm{x}) = \dfrac{k}{|\mathrm{i} - \mathrm{x}|^2}$

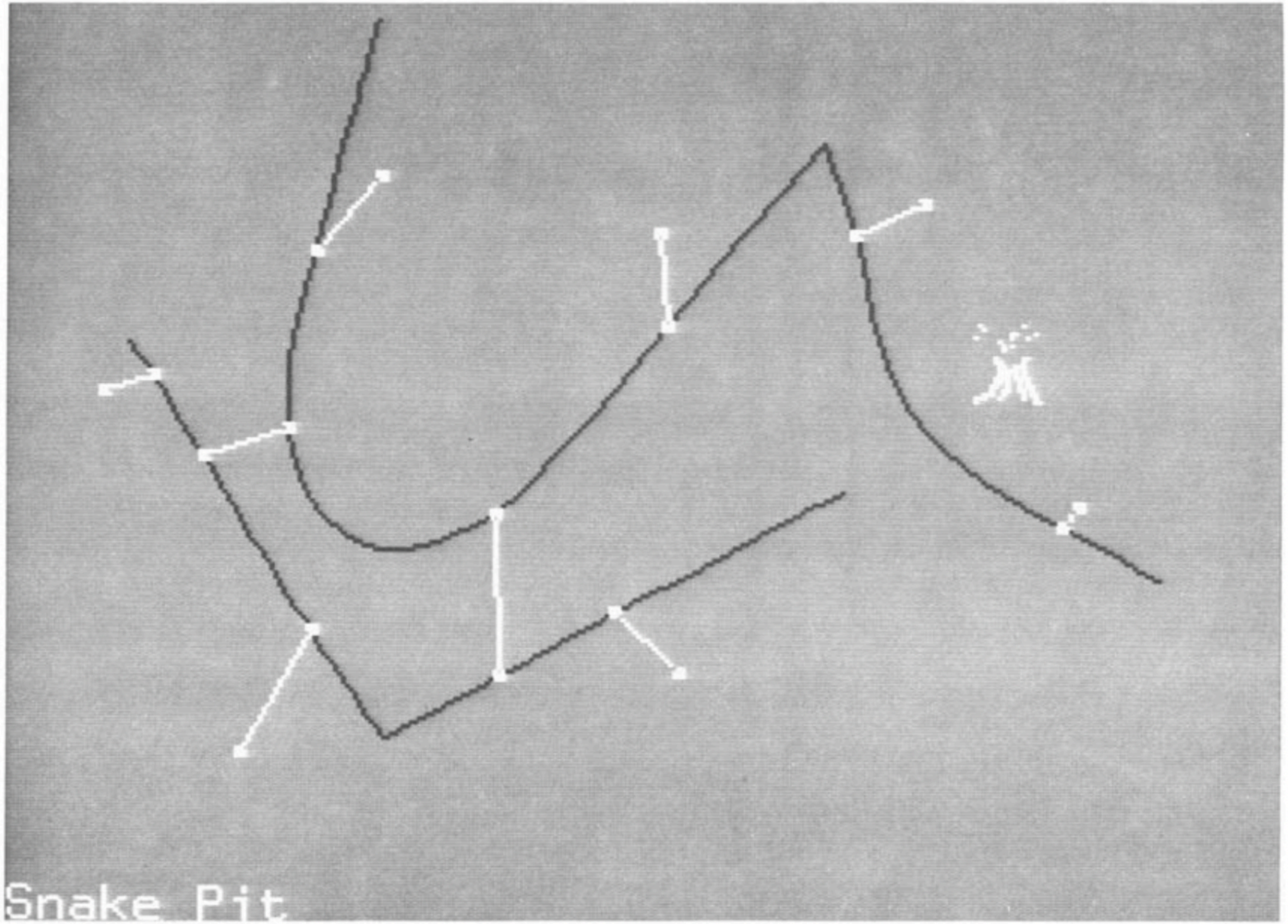


Fig. 2. The Snake Pit user-interface. Snakes are shown in black, springs and the volcano are in white.

Image Energy

- Image (Potential) Forces
- $E_{image} = w_{line}E_{line} + w_{edge}E_{edge} + w_{term}E_{term}$
- $E_{line} = I(x, y)$ - light lines or dark lines
- $E_{edge} = -\,|\nabla G(x, y) * I(x, y)|^2$
- $E_{term} = \dfrac{C_{yy}C_x^2 - 2C_{xy}C_xC_y + C_{xx}C_y^2}{(C_x^2 + C_y^2)^{3/2}}$ and C is the smoothed image

Image Energy

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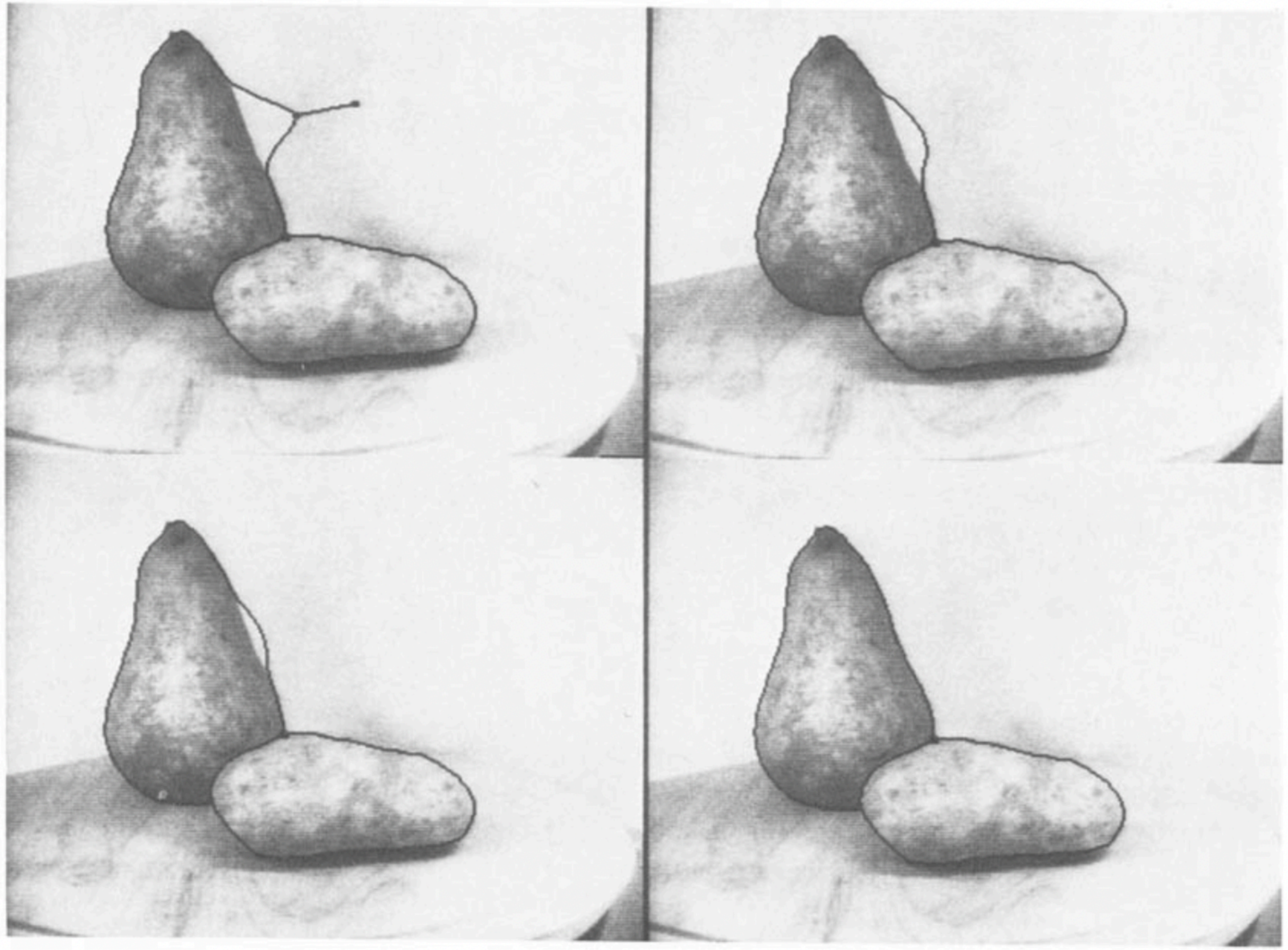

Fig. 3. Two edge snakes on a pear and potato. Upper-left: The user has pulled one of the snakes away from the edge of the pear. Others: After the user lets go, the snake snaps back to the edge of the pear.

Minimizing the energy

- Gradient descent
- The Euler-Lagrange Equation

Euler-Lagrange

- We want to minimize $\int E(s, \gamma, \gamma', \gamma'')ds$
- By Euler-Lagrange method, we need to solve
- $\frac{\partial E}{\partial \gamma} - \frac{\partial}{\partial s}\frac{\partial E}{\partial \gamma'} + \frac{\partial^2}{\partial s^2}\frac{\partial E}{\partial \gamma''} = 0$

Discrete implementation

$$\mathbf{D_2} = \begin{bmatrix} -2 & 1 & 0 & 0 & \cdots & 0 & 0 & 1 \\ 1 & -2 & 1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 & -2 & 1 \\ 1 & 0 & 0 & 0 & \cdots & 0 & 1 & -2 \end{bmatrix}$$

$$\mathbf{D_4} = \begin{bmatrix} 6 & -4 & -1 & 0 & \cdots & 0 & 1 & -4 \\ -4 & 6 & -4 & 1 & \cdots & 0 & 0 & 1 \\ 1 & -4 & 6 & -4 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 1 & 0 & 0 & 0 & \cdots & -4 & 6 & -4 \\ -4 & 1 & 0 & 0 & \cdots & 1 & -4 & 6 \end{bmatrix}$$

Code demo

Reference

Kass, Michael, Andrew Witkin, and Demetri Terzopoulos. "Snakes: Active contour models." International journal of computer vision 1.4 (1988): 321-331.

Sapiro, Guillermo. Geometric partial differential equations and image analysis. Cambridge university press, 2006.

Ivins, Jim, and John Porrill. "Everything you always wanted to know about snakes (but were afraid to ask)." *Artificial Intelligence* 2000 (1995).