LIE GROUPS, ASSIGNMENT 3, DUE OCTOBER 26

(1) Find the character table for the group S_4.
(2) Let G be any group. Recall that we defined a map $Rep(G) \to \mathbb{C}_{cl}[G]$.
    (a) Prove that $Rep(\mathbb{Z})$ is a free vector space with basis $\mathbb{C} \setminus \{0\}$.
    (b) Prove that $Rep(\mathbb{Z}) \to \mathbb{C}_{cl}[G]$ is not injective.
    (c) Can you find something in the kernel?
(3) Let G be a finite group and H be a proper subgroup.
    (a) Prove that there exists a conjugacy class $S \subset G$ such that $S \cap H = \emptyset$.
    (b) Prove that $Rep(G) \to Rep(H)$ is not injective.
(4) Let G be a finite or compact Lie group and V be an irreducible representation.
    (a) Prove that the space of G-invariant bilinear forms on V is at most 1 dimensional.
    (b) Prove that if V carries a non-zero G-invariant bilinear form, then it must be non-degenerate and must be either symmetric or skew-symmetric.
    (c) V is said to be of **real type** if it carries a G-invariant symmetric bilinear form. Prove that V is of real type iff $V = W_{\mathbb{C}}$ for some real representation W.
(5) Let G be a finite or compact Lie group. Assume that every element of G is conjugate to its inverse. Prove that if V is any representation of G, then $V \cong V^*$.
(6) Find all irreps of $SO(3)$. (In class we found all irreps of $SU(2)$.)