

Mathematics 189-133B, Winter 2003
Vectors, Matrices and Geometry
Written Assignment 5, due in class, March 7, 2003

1. Suppose that A and B are $n \times n$ matrices.
 - (a) Show that if $AB = BA$, then $(AB)^2 = A^2B^2$.
 - (b) Show that if A and B are invertible and $(AB)^2 = A^2B^2$, then $AB = BA$.
 - (c) Let $A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$. Show that $(AB)^2 = A^2B^2$, but $AB \neq BA$.
 - (d) Find an example where $(AB)^2 = A^2B^2$, $AB \neq BA$ and A is invertible.
2.
 - (a) Let $V = \text{span}\{\vec{v}_1, \dots, \vec{v}_k\}$ be a subspace of $\mathcal{R}^n$, and suppose that $\{\vec{w}_1, \dots, \vec{w}_\ell\}$ is an independent subset of V . Show that $\ell \leq k$.
 - (b) Use this to show that any two bases for V have the same number of elements.