Mathematics 189-133B, Winter 2003
Vectors, Matrices and Geometry
Written Assignment 8, due in class, March 28, 2003

Suppose that $T : \mathcal{R}^n \longrightarrow \mathcal{R}^n$ is a linear operator. We define the *kernel of* T, as $ker(T) = \{\vec{v} \in \mathcal{R}^n : T\vec{v} = \vec{0}\}$. [In case $T = T_A$, this is just the null space of A.]

1. Show that $ker(T^k) \leq ker(T^{k+1})$ for any natural number k.
2. Show that there is an integer $k \leq n$ such that $ker(T^k) = ker(T^{k+1})$.
3. Give examples, for $n = 4$, to show that it is possible that
   (a) $T \neq 0$, but $ker(T) = ker(T^2)$.
   (b) $ker(T) \neq ker(T^2)$, but $ker(T^2) = ker(T^3)$.
   (c) $ker(T^2) \neq ker(T^3)$, but $ker(T^3) = ker(T^4)$.
   (d) $ker(T^3) \neq ker(T^4)$, but $ker(T^4) = ker(T^5)$.