Mathematics 189-133B, Winter 2003
Vectors, Matrices and Geometry
Written Assignment 9, due in class, April 4, 2003

Suppose that A is an $n \times n$ matrix over the reals and that $W \leq \mathcal{R}^n$ is a subspace. We say that W is A-*invariant* if, whenever $\vec{w} \in W$, it must also be the case that $A\vec{w} \in W$.

(In-your-head-question. Convince yourself that, no matter what A is, the trivial subspace $\{\vec{0}\}$ is A-invariant, and so is $\mathcal{R}^n$ itself. This part of the assignment will also be marked in your head, and the mark recorded there. So you don't need to hand it in.)

1. Suppose that A is a scalar matrix, i.e., $A = \lambda I$ for some scalar λ. Show that every subspace $W \leq \mathcal{R}^n$ is A-invariant. (So if $n \geq 2$ there are infinitely many A-invariant subspaces.)

2. Suppose instead that A is 2×2 and it has distinct eigenvalues $\lambda_1 \neq \lambda_2$. Show that, besides the trivial subspace and $\mathcal{R}^2$, the only other A-invariant subspaces are the eigenspaces.