Mathematics 189-133B, Winter 2003
Vectors, Matrices and Geometry
Written Assignment 1, due in class, Friday, January 24, 2003

1. Let Π_1 be the plane in R^3 defined by $\vec{n}_1 \cdot (\vec{x} - \vec{x}_0) = 0$. (So $\vec{n}_1$ is a normal vector and $\vec{x}_0$ some particular point on Π_1.) Let Π_2 be a different plane through the same point $\vec{x}_0$, with equation $\vec{n}_2 \cdot (\vec{x} - \vec{x}_0) = 0$. Show that the intersection of the two planes is the line given by the equation $\vec{x} = \vec{x}_0 + t(\vec{n}_1 \times \vec{n}_2)$.

2. Let A, B, C and D be any points in the plane that determine a quadrilateral $ABCD$. (We assume that these are listed in an order so that the diagonals are $\overline{AC}$ and $\overline{BD}$, and they cross.) Suppose that E, F, G and H are the midpoints of $\overline{AB}$, $\overline{BC}$, $\overline{CD}$ and $\overline{DA}$ respectively. Using vector methods, show that

   (a) The quadrilateral $EFGH$ is a parallelogram.

   (b) Assuming that $ABCD$ is itself a parallelogram, show that $EFGH$ is a rectangle if and only if $ABCD$ is a rhombus.

   (c) Assuming that $ABCD$ is itself a parallelogram, show that $EFGH$ is a rhombus if and only if $ABCD$ is a rectangle.