Mathematics 189-133B, Winter 2003
Vectors, Matrices and Geometry
Written Assignment 2, due in class, Friday, January 31, 2003

The *convex hull* of a set $\{P_1, P_2, \dots, P_k\}$ of points in R^3 is the set of all points Q such that $\vec{q} = c_1\vec{p}_1 + c_2\vec{p}_2 + \cdots + c_k\vec{p}_k$ for some scalars $c_1, \dots c_k$ that are ≥ 0 and add up to 1. So $(2,0,4) = \frac{1}{2}(2,-2,2) + \frac{1}{3}(3,0,6) + \frac{1}{6}(0,6,6)$ is in the convex hull of $\{(2,-2,2),(3,0,6,),(0,6,6)\}$, since $\frac{1}{2}+\frac{1}{3}+\frac{1}{6} = 1$. (Here $\vec{p}_j$ is the vector $\vec{OP}_j$ and $\vec{q}$ is $\vec{OQ}$.)

1. Show that the convex hull of a pair $\{P_1, P_2\}$ of points is the line segment $\overline{P_1P_2}$.
2. Show that, if P_1, P_2 and P_3 are not collinear, then the convex hull of $\{P_1, P_2, P_3\}$ is the set of all points Q on or inside the triangle $\triangle P_1P_2P_3$.

[Hint for (b). Notice that Q is on or in $\triangle P_1P_2P_3$ if and only if there is a point A on $\overline{P_1P_2}$ such that Q is on $\overline{AP_3}$.]