Mathematics 189-133B, Winter 2003
Vectors, Matrices and Geometry
Written Assignment 4, due in class, Friday, February 21, 2003

Let $p(x) = \sum_{j=0}^{k} a_j x^j = a_0 + a_1 x + \cdots + a_k x^k$ be a polynomial with real coefficients, and A an $n \times n$ matrix. We define $p(A) = \sum_{j=0}^{k} a_j A^j = a_0 I + a_1 A + \cdots + a_k A^k$.

1. Show that if there is an invertible matrix P such that $B = P^{-1}AP$, then $P^{-1}p(A)P = p(B)$. [Hint: first show that $P^{-1}A^j A = B^j$.]

2. Show that, if $p(A) = 0$, but $a_0 \neq 0$, then A is invertible. In fact, show that there is a polynomial $q(x)$ such that $q(A)A = I$.