Department of Mathematics and Statistics

MATH 222: Calculus III

Written Assignment 1

Due in class Friday October 4, 2002

Justify all of your assertions

1. (a) Determine whether the following series converge or diverge.

$$\text{(i)} \quad \sum_{n=0}^{\infty} \frac{\cos(n\pi)}{\sqrt{n+1}}, \qquad \text{(ii)} \quad \sum_{n=1}^{\infty} \frac{n(n+6)}{(n+1)(n+3)(n+5)}.$$

   (b) Find the values of p for which the series

$$\sum_{n=1}^{\infty} \frac{\ln n}{n^p}$$

   is convergent.

2. (a) Let

$$f(x) = \sum_{n=1}^{\infty} \frac{(x-3)^n}{5^n \sqrt{n^3}},$$

   defined on the interval of convergence I of the series. Find I and the radius of convergence R of the series.

   (b) Find a power series representation of $f'(x)$. For which values of x is this representation valid?

3. (a) Expand

$$f(x) = \frac{x + x^2}{(1-x)^3}$$

   as a power series. [Hint: Write $f(x)$ as $\frac{x}{(1-x)^3} + \frac{x^2}{(1-x)^3}$.]

   (b) Use part (a) to find the sum of the series

$$\sum_{n=1}^{\infty} \frac{n^2}{2^n}.$$

4. Given the curve C: $\vec{r}(t) = (t+1, \frac{2}{3}\sqrt{2}t^{\frac{3}{2}} + 362, \frac{1}{2}t^2)$, $(t > 0)$, find

   (a) the equation of the tangent line to C at $t = 1$;

   (b) the arc length of C between $t = 1$ and $t = 2$.

Estimated time for completion of this assignment is 90 minutes.