McGill University

MATH 222: Caculus III

Written Assignment 2

Due in class Friday November 1, 2002

JUSTIFY ALL YOUR ASSERTIONS

1. Given the curve $x = t^3/3,\ y = 2t,\ z = 2/t,\ (t > 0)$, find

   (a) the unit tangent, principal normal and curvature at any point;

   (b) the equation of the tangent line to the curve at the point where $t = 1$.

2. If $u = f(r)$, where f is differentiable and $r = \sqrt{x^2 + y^2 + z^2}$, show that, for $(x, y, z) \neq (0, 0, 0)$,

   $$(a)\ \ (\frac{du}{dr})^2 = (\frac{\partial u}{\partial x})^2 + (\frac{\partial u}{\partial y})^2 + (\frac{\partial u}{\partial z})^2 \qquad \text{and} \qquad (b)\ \ \nabla u = \frac{1}{r}\frac{du}{dr}(x\vec{i} + y\vec{j} + z\vec{k}).$$

3. (a) Find the equation of the tangent plane and normal line to the surface $z = 3xe^y - x^3 - e^{3y}$ at the point $(0, 0, -1)$.

   (b) Find the equation of the tangent plane to the surface

   $$x = 2s^2 + t^3, \quad y = s^2t^3, \quad z = s^2 - st^3$$

   at the point where $s = t = 1$.

4. Suppose that $T(x, y, z) = x^3y + y^3z + z^3x$ is the temperature at the point (x, y, z) in 3-space.

   (a) Calculate the directional derivative of T at the point $P(2, -1, 0)$ in the direction from P to the point $Q(1, 1, 2)$.

   (b) A mosquito is flying through space with constant speed 5 in the direction of increasing temperature. If the mosquito's direction of flight at any given point is always normal (perpendicular) to the level surface of $f(x, y, z) = 2x^2 + 3y^2 + z^2$ passing through this point, find the rate of change of temperature experienced by the mosquito when it is at the point $(2, -1, 0)$.