Geometric group theory, homework 3.

Problem 1. Find the Cayley graph of the group

(i) $\langle s,t|s^3=t^3=(st)^3=1\rangle$,

(ii) $\langle s,t,r|s^2=t^2=r^2=(st)^3=(tr)^3=(rs)^3=1\rangle$.

Problem 2. Find the Cayley graph of

(i) the symmetric group S_3 of permutations of $\{1,2,3\}$ with generators $(1,2),(2,3)$,

(ii) the symmetric group S_4 of permutations of $\{1,2,3,4\}$ with generators $(1,2),(2,3),(3,4)$,

(iii) the Heisenberg group $\langle s,t,r|[s,t]=[s,r]=1,\ [t,r]=s\rangle$.

Definition. An isometry g of a tree is

- *elliptic* if g fixes a point,
- *hyperbolic* if g preserves and translates along an isometrically embedded line (called the *axis of* g).

Problem 3. Show that each isometry g of a tree is elliptic or hyperbolic. Hint: consider the set of vertices v with minimal translation length $|v,g(v)|$.

Problem 4. Show that each finite group has property (FA).

Problem 5. Show that if a finite index subgroup F of G has property (FA), then G has property (FA) as well.

Problem 6. Show that each finite family of pairwise intersecting subtrees of a tree has a point in common.

Problem 7. Show that if a finitely generated group $G=\langle S\rangle$ acts on a tree and the elements s,st for all $s,t\in S$ are elliptic, then G has a global fixed point. Hint: use Problem 6.

Problem 8. Show that the groups from Problem 1 have property (FA), but admit subgroups of finite index without (FA).