Geometric group theory
Requirements for the final exam

1. The definition of the free product of groups. The proof of Gruško's Theorem.
2. The definition of a free product with amalgamation and its Bass–Serre tree. The statement of Serre's Theorem characterising groups with property (FA). The definition of a graph of groups and its fundamental group.
3. The definition of the Cayley graph and the Cayley complex. The definition of the hyperbolic plane $\mathbf{H}^2$ and different types of its isometries. The proof of Gauss–Bonnet Theorem for geodesic polygons in $\mathbf{H}^2$. The description of the action of $\mathrm{PSL}(2, \mathbf{Z})$ on the Farey graph.
4. The proof of the isoperimetric inequality for the tiling of $\mathbf{H}^2$ by regular octagons with angle $\frac{\pi}{4}$. The solution to the Word Problem for the closed genus 2 surface group using Dehn's Algorithm.
5. Definitions of small cancellation conditions. The proof that Dehn's Algorithm gives the correct output for $C'(\frac{1}{6})$ groups.
6. The definition of a δ–hyperbolic metric space, a Gromov hyperbolic group and Gromov product $(x|y)_w$. The proof of the theorem that Dehn's Algorithm gives the correct output for appropriate presentation of a Gromov hyperbolic group.
7. The definition of a quasi-isometry, and of a geometric action. The proof of Milnor–Švarc Theorem. The proof of the lemma on quasi-geodesic stability in δ–hyperbolic metric spaces. The definition of an undistorted subgroup and the proof that cyclic subgroups of hyperbolic groups are undistorted.
8. The definition of Gromov boundary and Gromov metric on the boundary. The proof of the proposition that it is indeed a metric. The statement of the Tits Alternative for Gromov hyperbolic groups.
9. The definition of the space *Ends(X)* of ends of a topological space X. The proof of the theorem that a finitely generated group has 0,1,2 or ∞ many ends. The statement of Stallings' Theorem and its proof for finitely presented groups.
10. The statement of Gromov's Theorem on groups with polynomial growth. The definition of the asymptotic cone of a metric space. The proof of the local compactness and the finite dimensionality of an appropriate cone for groups with polynomial growth.