MATH 599 Nonpositive Curvature
Problem list 4

Problem 1. Let C_r be a circle in the hyperbolic plane $\mathbb{H}^2$ of radius r, i.e. the set of points at distance r from a given point. Compute the length of C_r. Do not use the lemma from class on polar coordinates — this problem completes its proof.

Problem 2. Show that for any $r > 0$ the function $x \to \frac{\sinh(rx)}{rx}$ is increasing on $\mathbb{R}_+$. (We used this to show that $\mathrm{CAT}(\kappa)$ implies $\mathrm{CAT}(\kappa')$ for $0 > \kappa' > \kappa$.)

Problem 3. Show that the action of $\mathbf{SL}(n, \mathbb{R})$ on $P(n, \mathbb{R})_1$ is *transitive* meaning that for every pair of points $p, p' \in P(n, \mathbb{R})_1$ there is $g \in \mathbf{SL}(n, \mathbb{R})$ such that $g(p) = p'$.

Problem 4. In the action of $\mathbf{SL}(n, \mathbb{R})$ on $P(n, \mathbb{R})_1$, compute the *stabiliser* of each point $p \in P(n, \mathbb{R})_1$, that is, the set of $g \in \mathbf{SL}(n, \mathbb{R})$ such that $g(p) = p$.

Problem 5. Let $A \subset P(n, \mathbb{R})_1$ be diagonal matrices (with entries positive and product 1). Describe A as a metric space (with the restriction of the Riemannian metric from $P(n, \mathbb{R})_1$).

Problem 6. Prove that $P(2, \mathbb{R})_1$ is isometric with rescaled $\mathbb{H}^2$. What is the scaling factor?