McGill University
Department of Mathematics and Statistics
MATH 599/706: Metric nonpositive curvature
Fall 2023

Instructor: Piotr Przytycki
Office: 927 Burnside Hall
Email: piotr.przytycki@mcgill.ca
Office hours: by appointment

Textbook: Martin Bridson and André Haefliger *Metric spaces of non-positive curvature*

Syllabus: This will be a course on curvature defined in metric terms instead of differential-geometric. We will start with studying a handful of examples: hyperbolic geometry, complex hyperbolic geometry, symmetric spaces. We will give some motivation from differential geometry but we will introduce the spaces of metric non-positive curvature using a purely metric condition. We will discuss singular examples of such spaces such as trees or polyhedral complexes, in particular cube complexes. We will prove the Cartan-Hadamard theorem saying that if a simply connected metric space is locally nonpositively curved, then it is globally nonpositively curved.
We will discuss isometries and groups of isometries acting on such spaces. We will prove the flat torus theorem, the fixed-point theorem for finite subgroups, and we will discuss the decision problems such as the word problem and the conjugacy problem. We will define the boundary at infinity of such a space. We will discuss further examples such as Coxeter groups and buildings and revisit the symmetric spaces. Finally we will mention a construction using complexes of groups.

Half of the class time will be devoted to the lecture per se. The other half will be devoted to solving together the weekly problems from the homework assignment (they will not be due in writing, but students will be expected to show the solutions on board during the class).

Assessement:
75% final exam from the proofs.
25% two **in-class** midterms, on **October 12** and **November 21**, from the problems similar to homework assignments, where solving at least half of the midterm problems suffices to obtain the maximal score.
The final grade will be increased for students presenting regularly good solutions of homework assignments.

Academic Integrity: McGill University values academic integrity. Therefore all students must understand the meaning and consequences of cheating, plagiarism and other academic offences under the Code of Student Conduct and Disciplinary Procedures (see http://www.mcgill.ca/integrity/ for more information).

Language Policy: In accord with McGill University's Charter of Students' Rights, students in this course have the right to submit in English or in French any written work that is to be graded.

Extraordinary Circumstances: In the event of extraordinary circumstances beyond the University's control, the content and/or evaluation scheme in this course is subject to change.