Instructor: Dr. R.A.G. Seely — Test 2

(April 2016) — (version A)

Calculus III (Maths 201–DDB)

(Marks)

Justify *all your answers—just having the correct answer is not sufficient.*
Pace yourself—a rough guide is to spend not more than 2m minutes or so on a question worth m marks.
Remember vectors are given in **boldface***: so* $\boldsymbol{v}$ *is a vector,* v *is a scalar.*

(3) 1. Prove that $|\boldsymbol{u}+\boldsymbol{v}|^2+|\boldsymbol{u}-\boldsymbol{v}|^2 = 2(|\boldsymbol{u}|^2+|\boldsymbol{v}|^2)$, for any vectors $\boldsymbol{u}, \boldsymbol{v}$. *(Hint: dot product!)*

(5) 2. (a) Is the following statement true or false?: If $\boldsymbol{u}, \boldsymbol{v}$ are unit vectors, then $\boldsymbol{u}\times\boldsymbol{v}$ is a unit vector also. If true, prove it; if false, give an example where it is false.

(b) Suppose that an object moves along a curve $\boldsymbol{r}=\boldsymbol{r}(t)$ so that for all t, the velocity and acceleration $\boldsymbol{v}$, $\boldsymbol{a}$ are always unit vectors. What is the curvature κ of the path of the curve? *(The justification for your answer is what I shall be marking, not merely the answer, which you might think is obvious!)*

(Hint: for questions 1,2 don't use components; instead use the general properties of vector functions.)

(6) 3. Name and sketch the following surfaces in 3-space. Show all your work, including traces and intercepts.

(a) $z=\sqrt{y^2-x^2-1}$ (b) $\rho = 9\cos\varphi$

4. A particle moves along the space curve $\boldsymbol{r} = \langle\, \mathrm{e}^t\,,\ \mathrm{e}^t\sin t\,,\ \mathrm{e}^t\cos t\,\rangle$.

(2) (a) Show that this curve lies on the surface of a cone. *(Hint: Pythagoras)*

(4) (b) Sketch the graphs of the cone and the space curve, indicating the orientation (direction of increasing t) of the curve.

(6) (c) Find the unit tangent vector $\boldsymbol{T}(t)$; the unit normal vector $\boldsymbol{N}(t)$; and the curvature $\kappa(t)$.

(4) (d) Find the tangential and normal components a_T, a_N of acceleration.

(3) (e) Find the parametric equations of the tangent line to the curve at the point where $t=0$.

(Hint: do the algebra carefully, and you will find that things simplify nicely.)

(4) 5. Find the equation of the tangent plane to the surface $z=\cos(x)+\sin(y)$ at the point $(0,0,1)$. Use the tangent plane to estimate the value of z at $(0.1,-0.1)$. *(You don't need a calculator for that, but you may use one to see how "good" the estimate is.)*

(9) 6. Calculate the following limits (if possible; if the limit does not exist, say so, and justify your answer).

(a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2y^3}{x^2+y^6}$ (b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy^3}{x^2+y^6}$ (c) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2+2}{y^6+6}$

(4) 7. Show that $z=x/y$ is a solution of the equation $x\dfrac{\partial z}{\partial x}+y\dfrac{\partial z}{\partial y}=0$.

(Total: 50)