



Cal II (S) (Maths 201–NYB)

(Marks)

Justify all your answers—just having the correct answer is not sufficient.Pace yourself—a rough guide is to spend not more than $2m$ minutes or so on a question worth m marks.

(39) 1. Evaluate the following:

(a) $\int x^5 \sqrt{x^3 - 1} dx$

(b) $\int x^2 \sqrt{x^3 - 1} dx$

(c) $\int \frac{dt}{\sqrt{4t^2 - 9}}$

(d) $\int_0^{1/2} x \arcsin x dx$

(e) $\int \frac{\tan t}{\sqrt{\sec t}} dt$

(f) $\int \sec^4(5\theta) \tan^4(5\theta) d\theta$

(g) $\int \frac{x^4 + 1}{x^3 + 2x^2 + x} dx$

(h) $\int \frac{\ln x dx}{\sqrt{x \ln x - x}}$

(i) $\int (\sin^2(t) + \cos^5(2t)) dt$

(j) $\int \frac{e^{1+\sqrt{x}}}{\sqrt{x}} dx$

(k) $\int \frac{\sqrt{1-x^2}}{x^2} dx$

(l) $\int e^x \sin 3x dx$

(m) $\int x^3 e^{4x} dx$

(3) 2. Given that $f(0) = 0$, $f(1) = 4$, $f'(0) = 3$, $f'(1) = 5$, what is $\int_0^1 x f''(x) dx$? (Hint: parts)(2) 3. Find the derivative $\frac{dy}{dx}$ for the function $y = \arctan\left(\frac{1}{1+x}\right)$. Simplify your answer.(1) 4. (a) Calculate the exact value of $\sin\left(\arctan\left(\frac{5}{12}\right)\right)$.(1) (b) Calculate the exact value of $\arctan\left(\sin\left(\frac{3\pi}{2}\right)\right)$.(2) (c) Simplify the expression $\tan(\arccos(x))$, expressing your answer in a form without any trig or inverse trig. Does the answer depend on whether x is positive or negative? (Justify your answer.)(2) (d) If θ is an angle for which $\tan \theta = -\frac{1}{3}$, then what are all the possible values of $\cos \theta$?
What are all the possible values of $\cos\left(\arctan\left(-\frac{1}{3}\right)\right)$?**Briefly** explain the connection between these two answers.

(Total: 50)