MATH 204 - ASSIGNMENT 3: SOLUTIONS

1. (a) Note that the critical values of the Chisquared(1) distribution for $\alpha = 0.05$ and $\alpha = 0.01$ are 3.841 and 6.635 respectively. Using the Chi-squared statistic for association in 2×2 tables:

   - Low Rate

   $$\text{Observed} : \begin{bmatrix} 20 & 1093 \\ 5 & 715 \end{bmatrix} \qquad \text{Expected} : \begin{bmatrix} 15.180 & 1097.820 \\ 9.820 & 710.180 \end{bmatrix}$$

   so that $X^2 = 3.950$. Hence the null hypothesis of independence between row and column classifications is **rejected** at $\alpha = 0.05$. The p-value is 0.047.

   3 Marks

   - High Rate

   $$\text{Observed} : \begin{bmatrix} 22 & 144 \\ 99 & 1421 \end{bmatrix} \qquad \text{Expected} : \begin{bmatrix} 11.913 & 154.087 \\ 109.087 & 1410.913 \end{bmatrix}$$

   so that $X^2 = 10.205$. Hence the null hypothesis of independence between row and column classifications is **rejected** at $\alpha = 0.05$ and $\alpha = 0.01$. The p-value is 0.001.

   3 Marks

   (b) After Pooling

   $$\text{Observed} : \begin{bmatrix} 42 & 1237 \\ 104 & 2316 \end{bmatrix} \qquad \text{Expected} : \begin{bmatrix} 53.065 & 1225.935 \\ 92.935 & 2147.065 \end{bmatrix}$$

   so that $X^2 = 3.781$. Hence the null hypothesis of independence between row and column classifications is **not rejected** at $\alpha = 0.05$. The p-value is 0.052.

   3 Marks

   (c) The following table contains the (estimated) odds ratio, log odds ratio, standard error of the log odds ratio and Z statistic which we compare with the standard normal distribution.

	$\widehat{\psi}$	$\log \widehat{\psi}$	s.e.	z
Low Rate	2.617	0.962	0.502	1.915
High Rate	2.193	0.785	0.251	3.123
Pooled	0.697	-0.360	0.186	-1.935

   The two-sided critical values for $\alpha = 0.05$ are ± 1.960, whereas the one-sided versions are ± 1.645 (from tables in McClave and Sincich). Hence we can conclude that for High Rate hospitals, there is a significant **positive** association, and for Low Rate hospitals there is a significant (against the one-sided alternative) **positive** association. However, for the Pooled data, there appears to be a significant (against the one-sided alternative) **negative** association. Here, a positive association corresponds to an increasing rate of UTI with increasing implementation of ABP policy, and vice-versa. Thus in the two strata (Low Rate and High Rate), it seems that there is evidence for a positive association, whereas overall there is evidence for a negative association.

   The conclusion we should make is that there is no reason why the pooled analyses should agree with the individual analyses. In general in this situation, we should report the stratified, and not the pooled results.

   4 Marks

   This result is known as ***Simpson's paradox****, and is the result of "***confounding***"; in simple terms, we are not allowed to pool data across the two tables.*

The apparent counterintuitive result in this data set for the two strata arises as the stratification is constructed on the basis of the rate of UTI infection, which is a variable that is also reflected in the counts in the table. Specifically, it might be that patients, because of their circumstances (type of operation, age etc), were already at high risk of UTI, and thus were pre-selected for ABP; thus the direction of causation is reversed. Note also that there is an imbalance in the rates of implementation of ABP policy: for the Low Rate hospitals the rate is 1113/1833 = 0.607, whereas for the High Rate hospitals, the rate is 166/1686 = 0.098. Hence it is possible that the effect of ABP policy implementation is **already being accounted for by the stratification**.

This is explained in detail in the paper from which the data are drawn, and which was cited in the question.

1 Mark

2. (a) The suitable parametric test is the one-way ANOVA F-test for the Completely Randomized Design. Either by hand, or from SPSS, the ANOVA table takes the following form:

ANOVA

relief

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	70581.778	2	35290.889	5.186	.019
Within Groups	102079.333	15	6805.289		
Total	172661.111	17			

Hence we **reject** the null hypothesis at $\alpha = 0.05$.

3 Marks

Note here that Levene's test indicates that the assumption of equal variances does not hold ($p = 0.005$), and the assumption of normality may be questionable.

(b) The suitable non-parametric testing procedure is the **Kruskal-Wallis** test.

	RB	Rank	R	Rank	U	Rank
	62	8	69	9	50	7
	74	10.5	43	6	-120	2
	86	12	100	17	100	17
	74	10.5	94	14	-288	1
	91	13	100	17	4	4
	37	5	98	15	-76	3
Sum		59		78		34

which yields the Kruskal-Wallis statistic $H = 5.725$. We compare this with the Chisquared(2) distribution, for which the $\alpha = 0.05$ critical value is 5.991. Hence we **do not reject** the null hypothesis of a common population median across the three groups.

3 Marks

Incidentally, the exact p-value computed using a permutation procedure is 0.0518, so again the null hypothesis is not rejected. This may be relevant if it is thought that the Chi-squared approximation to the null distribution may not be appropriate due to the relatively small sample size, or the presence of some ties.