Summary of Questions

MATH 222, Calculus 3 *Final Exam, Wednesday April 23, 2008*

1) Determine whether the following series are absolutely convergent, conditionally convergent or divergent. Justify your answers.

a)

$$\sum_{n=1}^{\infty}(-1)^n\frac{n^2}{n^2+1}.$$

b)

$$\sum_{n=1}^{\infty}(-1)^n\frac{\sqrt{n}}{n^2+1}.$$

c)

$$\sum_{n=1}^{\infty}(-1)^n\frac{n}{n^2+1}.$$

2) Find the Taylor series for the function

$$f(x)=\frac{1}{(2+x)^2}$$

at $a=1$ (i.e. in powers of $x-1$). Find the interval of convergence and determine convergence or divergence at the endpoints of the interval.
Hint: Work with an anti-derivative of $f(x)$.

3) Let

$$f(x)=\int_0^x \frac{1-cos(t)}{t^2}dt.$$

Find the Taylor polynomial $T_4(x)$ for $f(x)$ with center at 0.
Give and justify an estimate for $|R_4(\frac{1}{2})|$, where $R_4(x)=f(x)-T_4(x)$.

4) Let

$$f(x,y)=\frac{ln(1+x^2+y^2)}{x^2+y^2} \quad \text{for} \quad (x,y)\neq(0,0).$$

Is it possible to define $f(0,0)$ in such a way that f is continuous at $(0,0)$? If yes, what is $f(0,0)$?

5) Let L_1, L_2 be the lines given parametrically by

$$\mathbf{r}_1(t)=(1,0,0)+t(2,-1,0), \quad \mathbf{r}_2(t)=t(1,0,-1).$$

Find an equation of the plane parallel to L_1 and L_2 and equidistant from L_1 and L_2. What is this distance?

6) Explain why z is determined implicitly as a function $z=g(x,y)$ with $g(1,1)=1$ near the point $(1,1)$ by the relation

$$x^3+xyz+z^3=3.$$

Find $g_x,\ g_y,\ g_{xy}$ at $(1,1)$.

Please turn over page to see questions 7,8,9 and 10

7) Let the curve C be given as a vector function of time t by

$$\mathbf{r}(t) = e^{-t}(cos(t), sin(t)).$$

Find the curvature κ and the center of curvature for C at time t.

8) Let $f(x,y) = xy(x^2+y^2)$.

a) Show that $p = (0,0)$ is the only critical point of $f(x,y)$.
Show directly that p is neither a local maximum or a local minimum for $f(x,y)$. Explain why it is not possible to use the classification theorem for critical points to reach this conclusion.

b) Find the absolute maximum and minimum of $f(x,y)$ on the disc D of radius $\sqrt{2}$ with center at $(0,0)$.

Hint: Use the method of Lagrange multipliers on the boundary of D. Also, it will simplify your calculations if you make good use of the fact that x^2+y^2 is constant on the boundary.

9) Find the center of mass of the triangle bounded by the lines

$$x = 0, \quad y = 0, \quad x + y = 1$$

if the density at (x,y) is $\rho(x,y) = x + y$.

10) Let

$$f(x,y) = (1/\pi)e^{-\frac{x^2+y^2}{2}}.$$

Let D be the rectangle

$$\{(x,y) \mid |x| \le a,\ 0 \le y \le a\}.$$

Show that

$$1 - e^{-\frac{a^2}{2}} \le \iint_D f(x,y)dA \ \le 1 - e^{-a^2}.$$

Hint: Integrate $f(x,y)$ over suitable half-discs, one enclosed in and the other enclosing D.