MATH 222(Calculus III) Mid-Term Test
This is a mock (i.e., practice) exam!

1. Test the following series for convergence (absolute or conditional) or divergence.

(i) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+n}-\sqrt{n^2-1}}$, $\qquad$ $(ii)\displaystyle\sum_{n=1}^{\infty} \frac{\sin n}{n^2+1}\sqrt{n}$.

2. If $f(x) = \displaystyle\int_0^x \frac{1-e^{-t}}{t}dt$, then

(i) find a power series for $f(x)$ about $x = 0$;
(ii) find the interval of convergence of this power series;
(iii) compute $f(0.4)$ to four decimal place of accuracy justifying your answer.

3. Show that the points $A(-2,1,3)$, $B(2,3,0)$ and $C(-6,-1,6)$ are collinear.

4. (a) Find the point P of intersection of the two lines

$L_1 : x = 2t+3,\ y = -4t,\ z = t-3; \quad L_2 : x = 10s+3,\ y = 5s-25,\ z = -2s+4.$

(b) Determine the parametric equations of the line through the point P and the origin.

5. (a) Find a normal vector of the plane containing the points

$$P(1,-1,0),\ Q(2,1,-1),\ \text{and}\ R(-1,1,2).$$

(b) Find the area of the triangle formed by the above three points.

6. Find the equation of the plane Π which passes through the line of intersection of the two planes

$$\Pi_1 :\ x+y=2 \quad \text{and} \quad \Pi_2 :\ y-z=3,$$

and which is perpendicular to the plane $\Pi_3 :\ 2x+3y+4z=5$.

7. For the parametric curve $\mathbf{v}(t) = (t-\frac{t^3}{3}, t^2, t+\frac{t^3}{3})$, find

(1) the unit tangent and normal vectors $\mathbf{T}(t)$ and $\mathbf{N}(t)$ at any point;
(ii) the curvature $\kappa(t)$;
(iii) the length of the arc of the curve cut off between the planes $z = 0$ and $z = 12$.